Comparing poverty variations: A robustness assessment of the MDGs' achievements with respect to poverty alleviation

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Abstract

Has poverty been halved between 1990 and 2015? In the present paper, we show how well-know stochastic dominance tools can be used to check the robustness of claims regarding monetary poverty variations and then, using data from PovcalNet, provide a new picture of achievements with respect to poverty alleviation during the Millennium Development Goals' era. Using a sample of 90 developing countries, we notably observe that out of the 58 countries whose pace of poverty reduction was consistent with a 50% decrease of the headcount index over a 25-year period, 51 countries showed distribution changes that were in line with a more general conclusion that poverty would have been halved, whatever the poverty index we use, over the same period. Our results at the global level for the period 2002–2012 also show that the same conclusion robustly holds.

Keywords: Poverty comparisons, stochastic dominance, Millennium Development Goals.
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1 Introduction

The prominent target within the Millennium Development Goals (MDGs) officially adopted during the Millennium Summit of the United Nations in September 2000 was undoubtedly the very first one, namely the objective of halving, between 1990

* Université Clermont Auvergne, CNRS, IRD, CERDI, F-63000 Clermont-Ferrand, France; email: florent.bresson@uca.fr.

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and 2015, the share of the population living in extreme poverty. According to both the World Bank and the United Nations, the international community was successful with respect to this challenging objective, at least at the global level, probably before 2015. For instance, PovcalNet, the World Bank online tool for poverty measurement, reports that the value of the headcount index using their $1.90 poverty line fell from 35.96% in 1990 to 10.04% in 2015.[1] The ensuing Sustainable Development Goals do not carry on with such a relative target for global poverty reduction—the objective is extreme poverty eradication—, but the first goal also include a relative objective for national poverty alleviation, namely "by 2030 reduce at least by half the proportion of men, women and children of all ages living in poverty in all its dimensions according to national definitions." So targets regarding variations of the headcount index are still on the agenda.

However, it is widely acknowledged that the headcount index, in spite of its appealing simplicity, is a rather crude index with potentially non-desirable properties and so is outperformed from an ethical point of view by many other indices like those proposed by Watts (1968) or Foster et al. (1984). It then could be reasonable to define poverty variation objectives with respect to such indices, but the problem then becomes to choose one index within the bottomless set of admissible poverty indices. The choice of a specific value for the poverty line can also be a subject of endless discussions (see for instance the debates about the update of the international poverty line in Jolliffe and Prydz, 2015, Kakwani and Son, 2015, Klasen et al., 2015, Ferreira et al., 2015, World Bank, 2017) as it induces at least measurement errors and arbitrary choices. Besides, the empirical evidence shows that using a different poverty index or a different value for the poverty line for the comparison of two income distributions often results in a change in the poverty ordering between these two income distributions. Consequently, halving the share of the population living with less than the international poverty line does not necessarily mean that poverty has been halved in a broader sense.

In the present paper, we consider the validity of the claim "global poverty has been halved during the MDGs era" without reference to a specific poverty index. For that purpose, we propose an extension of the analytical framework initiated by Atkinson (1987) and based on the use of stochastic dominance techniques to test the robustness of poverty orderings. More specifically, we derive necessary and sufficient conditions for testing *i)* whether some (absolute) poverty change is β times lower than another (absolute) poverty change, and *ii)* whether poverty in one distribution is β times lower than poverty in an other distribution for various classes of poverty indices and different values of the poverty line. It also makes it possible to define "bounds of certainty" that delimit values of β such that a dominance relationship of a given order cannot be observed. In other words, it allows for instance to conclude

[1]These were the values reported on the website on the 3rd of July, 2020 (http://iresearch.worldbank.org/PovcalNet/).

that, over a given period, poverty has unambiguously changed by at least x percents, but for sure, no more than x' percents.

The structure of the paper is the following. Section 2 introduces notations and our extended stochastic dominance framework. In section 3, we assess to which extent the statement "poverty has been (at least) halved during the MDGs era" can be regarded as true using raw data provided by PovcalNet. The analysis is performed both at the global and the national levels. In particular, focusing on the subperiod 2002-2012, we show with the help of synthetic income distributions for 109 countries that accounted for approximately 82% of the World population in 2012 that extreme poverty was at least halved during this subperiod, a result that does not rely on the choice of a specific poverty index nor on a specific value for the poverty line. In addition, among a reduced sample of 90 countries for which the income distribution was sufficiently well described, we could observe that the pace of poverty reduction, using the headcount index and the international extreme poverty line, was consistent with the realization of MDGs target 1A in 55 cases. We show that this conclusion is robust in 51 cases, considering the set of monotone poverty indices, and in 60 cases, with the narrower set of poverty indices that comply with the strong versions of the transfer and transfer sensitivity axioms. Section 4 concludes.

2 NOTATIONS AND TOOLS

2.1 FRAMEWORK AND KNOWN RESULTS

Let y_i describe an individual i's attribute defined on the domain $\boldsymbol{K} := [\kappa^-, \kappa^+] \subset \Re$. For the sake of simplicity, y_i will generally be called income, but we may also consider consumption, wealth, or any relevant non-monetary attribute that can be described by a continuous variable. Whatever y_i refers to, a person's well-being is supposed to be a non-decreasing function of this attribute. Considering a population of n individuals, $\boldsymbol{y} := (y_1, \ldots y_n)$ is a n-vector of individuals income. The income distribution can alternatively be described using the cumulative distribution function $F : [\kappa^-, \kappa^+] \to [0, 1]$.

As noted in Sen (1976), poverty measurement is basically a two-step procedure. Firstly, we have to distinguish the poor from the non-poor, that is to choose a poverty line $z \in \boldsymbol{K} \setminus \{\kappa^-\}$ below which an individual is deemed poor and non-poor if its income is above. Here, this poverty line is exogenously defined, *i.e.* an absolute view of poverty is used for the present framework, and is the same for each person. Secondly, we assess the poverty level of each individual and aggregate the corresponding index over the whole population to get an estimate of the overall poverty level. A common practice is to focus on normalized additive poverty indices P that are the kernel of large classes of subgroup-consistent poverty indices. A poverty index is said to comply with subgroup-consistency if it provides orderings that are consistent with the orderings observed when comparing subgroups of the two populations under scrutiny. In other words, the overall level of poverty should not increase whenever poverty decreases within some subgroup of the population and is unchanged outside that group, assuming a constant structure of the population. Foster and Shorrocks (1991) have shown that every subgroup-consistent poverty index is an increasing transform of an additive poverty index of the type (1), so that poverty orderings observed with classes of additive poverty indices should also hold for the corresponding classes of subgroup-consistent poverty indices.

Considering the income distribution $\boldsymbol{y}$ and the poverty line z, a general expression for the poverty level $P_{\boldsymbol{y}}(z)$ is then:[2]

$$P_{\boldsymbol{y}}(z) := \int_{\kappa^-}^{\kappa^+} \pi(y,z)\, dF(y), \tag{1}$$

where $\pi : \boldsymbol{K} \times \boldsymbol{K} \backslash \{\kappa^-\} \to \Re_+$ is an individual poverty index that is at least piecewise continuous on $\boldsymbol{K}$, non-increasing in y_i, and such that:[3]

$$\pi(y,z) \begin{cases} \geq 0 & \text{if } y < z \\ = 0 & \text{otherwise} \end{cases}. \tag{2}$$

with $\pi(y,z) > 0$ for at least one income level in $[\kappa^-, z[$ so as to avoid degenerate poverty indices.

Since poverty indices are social indices, that is indices with a normative content, they should satisfy a certain number of additional properties that help defining an axiomatic framework for poverty measurement. For instance the second part of condition (2) is related to the normalization axiom that states that poverty is zero if no one in the population falls below the poverty line. More important, the constancy of π above the poverty line is a translation of the focus axiom according to which increasing the income of a non-poor person does not change the poverty level other things being equal (notably the poverty line). Another widely accepted axiom is the monotonicity axiom that, in its weakest form, imposes a poverty index not to increase after the increment of a poor person's income.[4] This defines a first class of poverty indices Π_0^1 thereafter called the class of monotone poverty indices. Indices from such classes as Π_0^1 are called canonical poverty indices in Foster and Shorrocks (1991). In this paper, they show that additive decomposability is observed for any increasing affine transform of a canonical index.

[2]In the present paper, all integrals are to be interpreted as Riemann-Stieltjes integrals. Conditions for computing by parts such integrals, even for piecewise functions, are always supposed to be satisfied.

[3] A function g is said to be piecewise continuous on $[a,b]$ if there exists a finite subdivision $\{x_0, x_1 \ldots x_m\}$ of $[a,b]$, where $x_0 = a$ and $x_m = b$, such that, $\forall k \in \{1, \ldots m\}$, the function g is continuous on $]x_{k-1}, x_k[$ and has finite limits on the right and left ends of the interval (Kaplan, 2002, p. 472). In the present paper, we suppose that discontinuities, if they exist, are all jump discontinuities.

[4]For a relatively comprehensive review of poverty measurement axioms, see Zheng (1997).

More formally:

$$\Pi_0^1 := \left\{ P \;\middle|\; \begin{array}{l} \pi \in \hat{\mathcal{C}}^1_{\boldsymbol{K}} \\ \pi(y+\varepsilon, z) - \pi(y, z) \leq 0 \quad \forall y + \varepsilon < z, \varepsilon > 0 \end{array} \right\}, \tag{3}$$

where $\hat{\mathcal{C}}^1_{[a,b]}$ denotes the set of piecewise smooth functions with respect to y on the interval $[a, b]$.[5]

This class of poverty indices notably includes the famous indices proposed by Watts (1968), Chakravarty (1983) and Foster et al. (1984) where the individual poverty index $\pi(y, z)$ is respectively defined on the poverty domain as $\log z - \log x$, $1 - \left(\frac{x}{z}\right)^\beta$ with $\beta \in]0, 1]$, and $\left(1 - \frac{x}{z}\right)^\alpha$ with $\alpha \geq 1$. However, an infinite number of rival indices from Π_0^1 are also likely to be proposed since there may be little agreement regarding the way we should evaluate the relative burden of poverty of two poor persons with different income levels. Social analysts then have a rich man's problem in choosing a specific poverty index since no index can be regarded a priori as superior to the other within our basic axiomatic framework. This affluence would not be a problem if any poverty ordering of two populations using a specific poverty measure from Π_0^1 was not likely to be reversed after turning to an alternative poverty index from the same class.

Poverty orderings may also crucially depend on the choice of the income value for the poverty line. Designing a poverty line is a difficult exercise that generally involves doing many arbitrary choices between different relevant techniques and approaches. Let alone the usual problems related to the obtaining of population estimates using sampling procedures, one cannot reasonably believe that the chosen value of income is the only admissible value for the poverty line as the definition of human needs and the appropriate cost of fulfilling them is a very difficult task. This contingency of poverty orderings to poverty line and poverty index choices is thus a serious issue for the study of poverty, in particular for monitoring poverty changes.

These problems were first addressed in Atkinson's (1987) seminal paper where a stochastic dominance approach was proposed to test the robustness of poverty orderings with respect to both the poverty index and the poverty line. Assuming that everyone agrees that the poverty line will never be lower than $z^- \in \boldsymbol{K} \setminus \{\kappa^+\}$ and never exceeds an upper bound $z^+ \in [z^-, \kappa^+]$, let $A \succcurlyeq_{0,1}^{z^-,z^+} B$ denote a situation where $P_{\boldsymbol{y}_A}(z) - P_{\boldsymbol{y}_B}(z)$ will never be positive whatever $P \in \Pi_0^1$ and $z \in [z^-, z^+]$. In the present paper, following Atkinson (1987), we consider weak orderings, hence the pos-

[5]A function g is said to be piecewise smooth on $[a, b]$ if g is piecewise continuous on $[a, b]$ (see footnote 3) and there exists a finite subdivision $\{x_0, x_1 \ldots x_m\}$ of $[a, b]$, where $x_0 = a$ and $x_m = b$, such that, $\forall k \in \{1, \ldots m\}$, g is continuously differentiable on $[x_{k-1}, x_k]$ where the derivative at x_{k-1} shall be understood as right-handed and the derivative at x_k shall be understood as left-handed (Kaplan, 2002, p. 472). It is worth stressing that this definition implies that g is not necessarily continuous at each x_k, contrary to the definition of piecewise differentiability that that is chosen by Duclos and Makdissi (2004). As a consequence, the first restriction in (3) means that this class of poverty indices does not necessarily comply with the restricted continuity axiom.

sibility of having $P_{\boldsymbol{y}_A}(z) = P_{\boldsymbol{y}_B}(z)$ whatever the chosen poverty index and the value of the poverty line. This differs notably from Zheng (1999) that proposes conditions so that $P_{\boldsymbol{y}_A}(z) > P_{\boldsymbol{y}_B}(z)$ is always true with the considered restrictions on π and z.

It can then be shown that $A \succcurlyeq_{0,1}^{z^-,z^+} B$ if and only if there is no $z \leq z^+$ such that the difference $P^1_{\boldsymbol{y}_A}(z) - P^1_{\boldsymbol{y}_B}(z)$ is strictly positive, where $P^s_{\boldsymbol{y}}(z)$ is:[6]

$$P^s_{\boldsymbol{y}}(z) := \frac{1}{n}\sum_{i=1}^{n}(z - \min\{y_i, z\})^{s-1}\,. \tag{4}$$

This stochastic dominance approach has later been discussed and refined in many papers, including Foster and Shorrocks (1988b,a), Atkinson (1992), Jenkins and Lambert (1993), Zheng (1999, 2000a,b), Duclos and Makdissi (2004, 2005) and Bresson (2014). The specificity of the approach is that it does not provide a complete order. If a dominance relationship is observed, then a robust ordering can be provided. Otherwise, the test is not conclusive and no robust conclusion can be proposed given the chosen analytical framework. In that case, the usual strategies are either to lower the upper bound z^+ until a first-order dominance relationship is observed or to introduce additional restrictions on the definition of admissible poverty indices so as to consider subsets of Π^1_0. In the present paper, we consider the second way of raising the ordering power.

For this purpose, we define the classes Π^s_r, $s \in \mathfrak{N} \setminus \{0,1\}$, $r \in \{0,\dots s-1\}$, of poverty indices defined as:[7]

$$\Pi^s_r := \left\{ P \;\middle|\; \begin{array}{l} \left.\frac{\partial^j \pi}{\partial y^j}\right|_{y=z} = 0 \quad \forall j \in \{0,\dots r-1\} \text{ if } r \geq 1 \\ \frac{\partial^j \pi}{\partial y^j} \in \mathcal{C}^0_{[\kappa^-,z[} \quad \forall j = 0,\dots s-2 \\ \frac{\partial^{s-1}\pi}{\partial y^{s-1}} \in \hat{\mathcal{C}}^1_{[\kappa^-,z[} \\ (-1)^j \left(\left.\frac{\partial^j \pi}{\partial y^j}\right|_{y=x+\varepsilon} - \left.\frac{\partial^j \pi}{\partial y^j}\right|_{y=x} \right) \geq 0,\ \forall x+\varepsilon < z, \varepsilon > 0,\ j = 0,\dots s-1 \end{array} \right\}, \tag{5}$$

where $\mathcal{C}^0_{[a,b]}$ denotes the class of continuous functions on the interval $[a,b]$.[8] Whatever

[6]This result was first obtained by Atkinson (1987, Condition 1A) for the subset of indices from Π^1_0 such that π is continuous and differentiable over $\boldsymbol{K}$. Zheng (1999) generalized this results by allowing π to be discontinuous at the poverty line. Duclos and Makdissi (2004) also extended Atkinson's result with the possibility of having π continuous over $\boldsymbol{K}$ but not differentiable for a finite number of values within the poverty domain. As shown by Proposition 1 (page 9) and considering the case $\boldsymbol{y}_{A'} = \boldsymbol{y}_{B'}$, the condition $P^1_{\boldsymbol{y}_A}(z) - P^1_{\boldsymbol{y}_B}(z) \leq 0\ \forall z \in [\kappa^-, z^+]$ implies $P_{\boldsymbol{y}_A}(z) - P_{\boldsymbol{y}_B}(z) \leq 0$ also for measures with jump discontinuities within the poverty domain or at the poverty line as long as they comply with weak monotonicity.

[7]We do not consider the case $r = s$ studied in Duclos and Makdissi (2004) as dominance conditions for that case do not differ from those observed for $r+1 = s$.

[8]In the case of indices that can be differentiated everywhere on $[\kappa^-, z[$ at the s-th order, the last condition of (5) is equivalent to $(-1)^j \left.\frac{\partial^j \pi}{\partial y^j}\right|_{y=x} > 0,\ \forall x < z,\ j = 1,\dots s$. If π^{s-1} shows a finite number m of distinct jump discontinuities at the disjoint points that define the set $\boldsymbol{X} := \{x_1, \dots x_m\} \subset]\kappa^-; z]$, this interpretation is only valid for $j = 1, \dots s-1$. Indeed, for $j = s$, the fourth and third conditions then have to be interpreted as $(-1)^s \left.\frac{\partial^s \pi}{\partial y^s}\right|_{y=x} \geq 0,\ \forall x \in [\kappa^-, z[\setminus \boldsymbol{X}$ and

$s \in \mathfrak{N} \setminus \{0\}$, it can easily be checked that $\Pi^s_{r+1} \subset \Pi^s_r$, $r \in \{0 \ldots s-2\}$, and $\Pi^{s+1}_r \subset \Pi^s_r$, $r \in \{0 \ldots s-1\}$.

Poverty indices from Π^2_0 constitute a subset of Π^1_0 and show individual poverty functions that are convex with respect to income on the poverty domain. They consequently satisfy restricted continuity as well as the weak version of the transfer principle, i.e. any sequence of progressive transfers from poor individuals should not result in increased poverty. Restricted continuity, that is continuity on the poverty domain but virtually not at the poverty line, has notably be justified by Bourguignon and Fields (1997). It can be argued that the existence of essential needs whose satisfaction is not a matter of degree but has a pure dichotomous nature, may also justify the presence of discontinuities within the poverty domain. However, such discontinuities are not consistent with the respect of the weak transfer axiom. That is why restricted continuity is imposed in Π^2_0 while it is not in Π^1_0.

The class Π^2_1 is a subset of Π^2_0 such that continuity at the poverty line is observed. Convexity of the individual poverty function is thus observed over the whole domain of definition of π and a strong version of the transfer principle is therefore endorsed: progressive transfers from non-poor persons to poor persons should also not result in a larger poverty level.

The last commonly invoked sets of poverty indices, Π^3_r, are subsets of Π^2_0 and bring together poverty indices that respect the axiom of transfer sensitivity (Kolm, 1976, Foster and Shorrocks, 1987). In the context of poverty measurement (Kakwani, 1980, Foster and Shorrocks, 1988b), transfer sensitivity means that a progressive transfer of a given amount between two poor persons showing a given income difference reduces poverty the more the lower is their initial income. Members from Π^3_r are thus associated with a marked aversion for extreme poverty as social returns to income increments decrease at a decreasing speed. The individual poverty function is still convex on the poverty domain but its curve becomes more bent as we consider poorer incomes. With Π^3_0, restricted continuity is assumed (since $\pi \in \mathcal{C}^0_{[\kappa^-,z[}$). Π^3_1 assumes continuity at the poverty line but the graph of the individual poverty function is likely to show a kink at the poverty line. Finally, the class Π^3_2 brings together indices such that π is also smooth at the poverty line.

It is worth pointing that, due to the first restriction in (5), popular indices are likely to be excluded from the considered class of poverty indices as a result of an increase in r. Indeed, such an index like the Watts index belongs to Π^3_1 but is not included in Π^3_2 because it is not smooth at the poverty line. As shown by Zheng (2000b) and Duclos and Makdissi (2004), disregarding “kinked” poverty indices is necessary in order to be able to propose ordering criteria that purely rely on third-order stochastic dominance conditions, hence raising significantly the ordering power in comparison to second-order stochastic dominance tests used for poverty orderings associated

$(-1)^{s-1}\left(\lim_{x\uparrow x_k} \left.\frac{\partial^{s-1}\pi}{\partial y^{s-1}}\right|_{y=x} - \lim_{x\downarrow x_k} \left.\frac{\partial^{s-1}\pi}{\partial y^{s-1}}\right|_{y=x}\right) \geq 0 \; \forall x \in \boldsymbol{X}$.

with Π_1^2.

Our point of view is that the exclusion of "kinked" poverty indices shall not be regarded as an issue of practical importance since we conjecture about the existence of smoothed version of such indices that consequently belong to Π_2^3. Consider for instance poverty indices of the form (1) with:

$$\pi(y,z) = \begin{cases} \log z - \log y & \text{if } y < z - \varepsilon \\ \frac{z+\varepsilon-y}{2\varepsilon}\left(\log\left(z + \frac{y-z+\varepsilon}{2}\right) - \log y\right) & \text{if } z-\varepsilon \leq y < z+\varepsilon \\ 0 & \text{if } y \geq z+\varepsilon \end{cases} \tag{6}$$

with $\varepsilon \in]0, z-\kappa^-[$. With this index, the upper bound of the poverty domain becomes $z+\varepsilon$, but we can note that its limit equals the Watts index as $\varepsilon \to 0$. Consequently, for very small values of ε, this smoothed version of the Watts index can hardly be distinguished from its original version with observed income distributions. Yet, it is included in Π_2^3 since it can be shown that $\frac{\partial^2\pi}{\partial y^2}$ is continuous everywhere. The same can be done for many other indices like the Chakravarty indices, assuming $\pi(y,z) = \frac{z+\varepsilon-y}{2\varepsilon}\hat{\pi}\left(y, z+\varepsilon\frac{y-z+\varepsilon}{2\varepsilon}\right)$, $\forall y \in [z-\varepsilon, z+\varepsilon[$, where $\hat{\pi}$ is the individual poverty index used for the "kinked" poverty index. As a result, our feeling is that the gain from the drop of "kinked" poverty indices is worth the gain in the ordering power due to the possibility of focusing on a unique order for stochastic dominance tests.

Finally, for larger values of s, the sensitivity of poverty indices to extreme forms of poverty increases in comparison with less harsh situations. Loosely speaking, the relative weight given to very low income increases, when compared with less poor income, as s is raised. As stressed later, dominance tests associated with statements $A \succcurlyeq_{r,s}^{z^+} B$ can be performed using the function P_j with $j = r+1, \ldots s$.[9] The narrower the set of poverty indices, that is the higher are r and s, the easier it becomes to perform robust poverty comparisons since the ordering power increases with the order of dominance as shown by Duclos and Makdissi (2004) for their classes of poverty indices.

It is worth pointing that the aforementioned studies only consider the ordinal properties of poverty indices, hence making it possible to consider any continuous increasing transform of P. This means for instance that the normalization axiom is not required for the type of orderings associated with stochastic dominance tests. Removing that property does not represent a considerable sacrifice as its normative content is poor (Zheng, 1997). Indeed, considering for instance $P'_{\boldsymbol{y}}(z) = 1 + 2P_{\boldsymbol{y}}(z)$ is totally benign when the objective is only to rank income distributions in terms of poverty. As a consequence, the aforementioned results on poverty orderings $A \succcurlyeq_{r,s}^{z^-,z^+}$

[9] This result is not shown here, but can easily be obtained from Proposition 1, page 9, assuming $\boldsymbol{y}_{A'} = \boldsymbol{y}_{B'}$.

B hold for the larger classes Ξ_r^s of subgroup-consistent poverty indices defined by:

$$\Xi_r^s := \Big\{\varphi\big(P(z)\big)|P(z) \in \Pi_r^s\Big\}, \tag{7}$$

with $\varphi : \Re_+ \rightarrow \Re$ being continuous and increasing. As an illustration of poverty indices that belong to Ξ_0^1 but are not included in Π_0^1, we can cite members from the second family of poverty indices suggested by Clark et al. (1981) and the cross-sectional version of the equally distributed equivalent gap proposed by Duclos et al. (2010). As stressed below, ordering poverty changes generally means discarding subgroup-consistent poverty indices that are not additively decomposable.

2.2 Robust comparisons of poverty changes

As argued in the introduction, the objective of poverty analysis may sometime be slightly more ambitious than a simple ordering of the poverty levels of two income distributions. In particular, we may be interested in the magnitude of poverty changes without having to resort to any specific poverty index. It can then easily be understood that this logically induces giving up the use of dominance tools in connection with such large sets of poverty indices as Ξ_r^s. Nevertheless, it can be shown that robust conclusions can be obtained regarding both the sign and the magnitude of poverty changes provided the focus is put on classes Π_r^s. For that purpose, we consider two forms of comparability we may want a poverty index to satisfy in order to consider the magnitude of poverty changes:

- *variation comparability* means that it is possible to order poverty differences between pairs of income distribution, *i.e.* $\big(P(\boldsymbol{y}_B, z) - P(\boldsymbol{y}_A, z)\big) - \big(P(\boldsymbol{y}_{B'}, z) - P(\boldsymbol{y}_{A'}, z)\big) > 0$ can be interpreted as "a move from distribution A to distribution B is associated with a larger variation of poverty than a move from distribution A' to distribution B'."

- *ratio comparability* means that it is possible to compare the ratio of poverty levels for any pair of income distribution with any positive real number, *i.e.* $P(\boldsymbol{y}_A, z) - \beta P(\boldsymbol{y}_B, z) > 0$, $\beta \in \Re_{++}$, can be interpreted as "distribution A shows at least as much as β times poverty as the distribution B."

Both forms of comparability can be simultaneously considered in order to get robust comparisons of poverty changes using the following result:

Proposition 1.

$$P(\boldsymbol{y}_B, z) - P(\boldsymbol{y}_A, z) \leq \beta\big(P(\boldsymbol{y}_{B'}, z) - P(\boldsymbol{y}_{A'}, z)\big) \quad \forall P \in \Pi_r^s,\ z \in [z^-, z^+], \tag{8}$$

iff $P^j(\boldsymbol{y}_B, z) - P^j(\boldsymbol{y}_A, z) \leq \beta\big(P^j(\boldsymbol{y}_{B'}, z) - P^j(\boldsymbol{y}_{A'}, z)\big)$

$$\forall z \in [z^-, z^+],\ j = r+1, \dots s \tag{9}$$

$$\textit{and} \quad P^s(\boldsymbol{y}_B, z) - P^s(\boldsymbol{y}_A, z) \leq \beta\big(P^s(\boldsymbol{y}_{B'}, z) - P^s(\boldsymbol{y}_{A'}, z)\big) \quad \forall z \leq z^+. \tag{10}$$

Proof. See appendix A. □

In the case $r + 1 = s$, condition (9) is necessarily observed if condition (10) is satisfied, and so can be neglected.

It can easily be seen that Proposition 1 generalizes previous results, notably those of Zheng (1999) and Duclos and Makdissi (2004). Indeed, assuming $\boldsymbol{y}_{A'} = \boldsymbol{y}_{B'}$, the comparison in (8) boils down to the usual poverty comparison between the distributions A and B. Within this framework, one then finds the dominance conditions proposed in Zheng (1999, Propositions 1–3, 5) for $(r = 0, s = 1)$, $(r = 0, s = 2)$, $(r = 1, s = 2)$, $(r = 1, s = 3)$, $(r = 2, s = 3)$ but considering here a larger set of indices than those whose function π is s times continuously differentiable with respect to income over the whole poverty domain. Duclos and Makdissi (2004) propose the use of dominance test corresponding to the case $r + 1 = s$, but assume continuity of the $s - 1$-th derivative of π with respect to income at the poverty line. This would correspond to classes the case $r = s$ in our setting. We do not consider these subsets in our setting as dominance conditions are exactly the same as for $r + 1 = s$. Finally, regarding the ordering of poverty levels, our proposition is more general since this unnecessary restriction is dropped.

Nevertheless, the main contribution in Proposition 1 is the use of stochastic dominance techniques for the ordering of poverty changes. Pure variation comparability is obtained for $\beta = 1$, while pure ratio comparability corresponds to the case $P(\boldsymbol{y}_A, z) = P(\boldsymbol{y}_{A'}, z) = 0$.[10]

In the case we are only interested with ratio comparability and considering $r = 0$ and $s = 1$, Proposition 1 can very easily be implemented. Indeed, it simply means comparing the cdf for distribution A with a scaled up or scaled down version of the cdf for distribution B, depending of the value of β. If the curve depicting the former is nowhere above the curve depicting the latter considering all values for the poverty line up to z^+, we can robustly conclude that poverty has decreased by at least $100(1-\beta)$ percent as a result of a move from distribution A to distribution B whatever the value of the poverty line (with a maximum value of z^+) and the functional form of the poverty index, assuming it complies with monotonicity in addition to additive decomposability, anonymity, and the focus and normalization axioms.

With the more general setting considered with comparison (8), if condition (10) is fulfilled, namely the difference in the cdf associated with distributions A and B is never strictly larger than β times the corresponding difference for the distributions A' and B' up to z^+, we can robustly conclude that the poverty change corresponding to a move from A to B is not larger than β times the poverty change referring to a

[10] We thank the anonymous referee that suggested bringing together the propositions used for a previous version of this paper and that separately addressed the case of variation comparability and ratio comparability. The resulting Proposition 1 is more general.

move from A' to B' whatever our choice of poverty index among the class of monotone poverty indices and the chosen value for the poverty line (within the interval $[\kappa^-, z^+]$). It is worth pointing that Proposition 1 does not impose any robust poverty ordering between any pair of distributions within the set $\{A, A', B, B'\}$. For instance, it is possible to consider the case where poverty increases as a result of a move from A' to B' while it is not possible to say anything robust regarding the comparison between A and B. In that situation, assuming (9) and (10) are satisfied, we would be able to conclude that if poverty had increased as a result from A to B, the increase would robustly be lower than the reported increase associated with the move from A' to B'.

Considering $r = 0$ and $s = 2$ increases the ordering power as long as $z^- > \kappa^-$. Indeed, for this class of poverty indices, one has to look at the sign of both $\big(P^1(\boldsymbol{y}_B, z) - P^1(\boldsymbol{y}_A, z)\big) - \beta\big(P^1(\boldsymbol{y}_{B'}, z) - P^1(\boldsymbol{y}_{A'}, z)\big)$ $\forall z \in [z^-, z^+]$ and $\big(P^2(\boldsymbol{y}_B, z) - P^2(\boldsymbol{y}_A, z)\big) - \beta\big(P^2(\boldsymbol{y}_{B'}, z) - P^2(\boldsymbol{y}_{A'}, z)\big)$ $\forall z \in [\kappa^-, z^+]$. If $z^- = \kappa^-$, then the second condition is not binding since the satisfaction of the first entails the satisfaction of the second. This mirrors the observation made by Zheng (1999, p.357) that, in the case of poverty level comparisons, the conditions for a robust ordering considering the class of poverty indices complying with the weak transfer axiom and restricted continuity collapse to conditions based on first order stochastic dominance tests as z^- tends to κ^+. On the contrary, if there is no uncertainty regarding the value of the poverty line (i.e. $z^- = z^+$), condition (9) simply means that the headcount ratio difference between distributions A and B at z^+ shall not be larger than β times the headcount ratio difference between A' and B' for the same value of the poverty line. Conditions (10) based on second-order dominance tests are then more decisive and the ordering power significantly increases. The increase is even more important if one moves to $r = 1$, whatever the chosen value for z^-, as there is no necessity to use the conditions based on first-order stochastic dominance tests to obtain a robust ordering of poverty variations any more. It is worth stressing that these remarks regarding the crucial role of z^- and r hold for larger values of s. Indeed, imposing more restrictions regarding the behaviour of the derivatives of the individual poverty function within the poverty domain, that is considering an increase in s, does not raise the ordering power as long as $z^+ = \kappa^-$ and π is not supposed to be continuous, since we are still sticked to the necessity of having conditions based on the comparison of the headcount ratio fulfilled.

As underlined earlier, Proposition 1 relates to classes of additively decomposable but cannot be extended to the corresponding broader classes of subgroup-consistent poverty indices. However, the evidence is that the vast majority of empirical studies related to poverty assessment only consider additively decomposable poverty measures of the form (1). Consequently, giving up the use of subgroup-consistent but non-additively decomposable poverty indices should not be regarded as a real sacrifice for both practitioners and policy makers.[11] Nevertheless, it does not mean that these indices could not be supported. For instance, if we assume that the social returns of fighting poverty decrease as aggregate poverty becomes lower, poverty changes should be tracked using convex transforms of members of Π_r^s. Conjecturing that such subgroup consistent indices could be worth scrutiny, we discuss here some specific situations were robust orderings of poverty changes can be extended to subgroup-consistent but non-additively-decomposable poverty indices.

For instance, it can easily be seen that our framework requires poverty indices to be cardinal, that is to provide uniquely up to positive affine transformations a complete preorder of income distributions with respect to poverty. As in the case of utility functions, values returned by the poverty index are of little significance per se for variation comparability, as we are only focusing on poverty changes. Thus, $P(\boldsymbol{y}_B, z) - P(\boldsymbol{y}_A, z) > \beta\big(P(\boldsymbol{y}_{B'}, z) - P(\boldsymbol{y}_{A'}, z)\big) \Leftrightarrow P'(\boldsymbol{y}_B, z) - P'(\boldsymbol{y}_A, z) > \beta\big(P'(\boldsymbol{y}_{B'}, z) - P'(\boldsymbol{y}_{A'}, z)\big)$ if and only if $P' = a + bP$, with $a \in \Re$ and $b \in \Re_{++}$. Hence, in general, variation comparability can only be considered for the subset of Ξ_r^s such that transformations φ are affine, that is the subset of subgroup consistent poverty indices whose variations are additively decomposable.

This cardinality requirement may be slacken if additional information regarding the ordering of poverty distributions is available. A first trivial case is when $B \succcurlyeq_{r,s}^{z^- z^+} A$ and $A' \succcurlyeq_{r,s}^{z^- z^+} B'$ are simultaneously observed, hence when we observe a robust decrease in poverty in the former situation and a robust increase in the latter situation. We then know that the ordering of $\big(P(\boldsymbol{y}_B, z) - P(\boldsymbol{y}_A, z)\big)$ and $\big(P(\boldsymbol{y}_{B'}, z) - P(\boldsymbol{y}_{A'}, z)\big)$ is robust for all indices from Ξ_r^s, since any monotonically increasing transform of P will preserve the ordering between each pair of poverty indices and so the sign of poverty changes. Testing conditions (9) and (10) is consequently useless in this case. More interesting are some of the remaining cases regarding the ordering between any pair of distribution within the set $\{A, A', B, B'\}$ along with the satisfaction of conditions (9) and (10). For instance, with $B \succcurlyeq_{r,s}^{z^- z^+} A \succcurlyeq_{r,s}^{z^- z^+} A'$, having the conditions of Proposition 1 satisfied means that the ordering of the poverty variations $\big(P(\boldsymbol{y}_B, z) - P(\boldsymbol{y}_A, z)\big)$ and $\big(P(\boldsymbol{y}_{B'}, z) - P(\boldsymbol{y}_{A'}, z)\big)$ is also robust for all members of Ξ_r^s such that φ is concave. On the other hand, in the case $A \succcurlyeq_{r,s}^{z^- z^+} A' \succcurlyeq_{r,s}^{z^- z^+} B'$, the result extends to members of Ξ_r^s such that φ is convex.

With pure ratio comparability, it is necessary to focus on poverty indices that comply with the normalization axiom, but convex and concave transforms of members of Π_r^s can be considered depending on the value of β. More specifically, if $\beta < 1$ and $P(\boldsymbol{y}_B, z) < \beta P(\boldsymbol{y}_{B'}, z)\ \forall P \in \Pi_r^s$, this result will also be true for members of Ξ_r^s such that φ is convex and $\varphi(0) = 0$. It can also be shown that, in the case $\beta > 1$ and

[11] As pointed by a referee, the question is why we should care about subgroup-consistent but non-additively-decomposable poverty indices? It is true that such indices are of little interest if one is only interested in the ordinal properties of a poverty index and, to the best of our knowledge, the desirability of specific members of Ξ_r^s that do not belong to Π_r^s has not been discussed in the literature yet.

$P(\boldsymbol{y}_B, z) > \beta P(\boldsymbol{y}_{B'}, z)\ \forall P \in \Pi_r^s$, the result also applies to members of Ξ_r^s such that φ is concave and $\varphi(0) = 0$.

2.3 Specific uses

The test proposed in Proposition 1 can be contrasted with those commonly used for growth "pro-poorness" tests and that also implicitly imply comparing poverty variations. As summarized by Duclos (2009), the rival approaches of the growth "propoorness" concept all mean comparing the observed poverty change with the one that would have been observed under a counterfactual scenario. From a practical point of view, this counterfactual scenario is obtained with the use of some transform $h : \boldsymbol{K}^n \to \boldsymbol{K}^n$ on the observed initial income distribution. With the absolute approach defended by Ravallion and Chen (2003), h is the identity function. A growth spell is then deemed "propoor" if poverty has decreased during the chosen period. With the relative approach supported by Kakwani and Pernia (2000), h scales up or down each elements of the initial income distribution according to the registered growth rate in mean income. A growth spell is then regarded as "pro-poor" if the observed poverty change is to be preferred from a social point of view that the change that would have occurred had growth been inequality preserving. In all cases, assuming $\boldsymbol{y}_A$ and $\boldsymbol{y}_B$ are respectively the initial and final income distributions, growth will be said to have been "propoor" if $P(\boldsymbol{y}_B, z) - P(\boldsymbol{y}_A, z) \leq P\big(h(\boldsymbol{y}_A), z\big) - P(\boldsymbol{y}_A, z)$. As $P(\boldsymbol{y}_A, z)$ appears on both sides of this equation, it can be dropped, hence simplifying the test to a traditional poverty ordering between $\boldsymbol{y}_B$ and $h(\boldsymbol{y}_A)$, but the point is that growth "propoorness" tests are a special case of the comparisons considered for Proposition 1 with $\beta = 1$, $\boldsymbol{y}_{B'} = h(\boldsymbol{y}_A)$, and $\boldsymbol{y}_{A'} = \boldsymbol{y}_A$. Another obvious use of Proposition 1 in the context of growth "propoorness" analysis corresponds to the case where $\boldsymbol{y}_A$ and $\boldsymbol{y}_{A'}$ are respectively replaced by $h(\boldsymbol{y}_A)$ and $h(\boldsymbol{y}_{A'})$. In this situation, conditions (9) and (10) can be used to test whether growth was more "propoor" as a result of a move from A to B in comparison with a move from A' to B'.

Here, we suggest that a different approach for monitoring the poverty effects of economic growth can be considered using a target $1 - \beta$ on the rate of poverty variation rather than a scenario h for individual income changes. Let γ the rate of growth over the period of interest and η_P^z be the value of the growth elasticity of poverty associated with the poverty index P and the poverty line z.[12] The product $\gamma\eta_P^z$ can be used a benchmark for the relative change in poverty against which the observed relative variation shall be compared. Then, we can consider the situation of pure

[12]We do not go into details regarding the way the value of η_P^z should be chosen but give here some insights about possible choices. Firstly, it is worth pointing that η_P^z is an elasticity of poverty with respect to mean income that can either refer to an inequality-preserving process or to a specific pattern of inequality changes. Secondly, the value may be estimated using different techniques (a point estimate using Kakwani's (1993) formulas, an arc elasticity using past changes, simulations or econometric models as in Ravallion, 2001, or Adams, 2004) or chosen on the basis of political criteria, i.e. poverty reduction and growth declared objectives.

ratio comparability, that is $P(\boldsymbol{y}_A, z) = P(\boldsymbol{y}_{A'}, z) = 0$, along with $\beta = 1 - \gamma\eta_P^z$ to test whether poverty reduction was larger or not than in the benchmark scenario. Using Proposition 1 with these specific choices, one can then check whether this result does not depend on the specific poverty index or the chosen value for the poverty line.

Finally, we can stress that Proposition 1 can also be used for the comparison of changes in contributions to aggregate poverty. Let's assume that the population can be decomposed into $m \geq 2$ mutually exclusive groups. The share of group $g \in \{1, \dots m\}$ in total population is denoted $\lambda_g \in]0, 1]$, with $\sum_{g=1}^{m} \lambda_g = 1$ and $\boldsymbol{y}^g$ is the observed income vector for the g-th group. With F^g denoting the corresponding cdf and using additively decomposable poverty indices, aggregate poverty can then be expressed as:

$$P(\boldsymbol{y}, z) := \sum_{g=1}^{m} \lambda_g \int_{\kappa^-}^{\kappa^+} \pi(y, z)\, dF^g(y), \tag{11}$$

and the contribution C_g of group g to $P(\boldsymbol{y}, z)$ is then simply:

$$C_g(\boldsymbol{y}, z) = \lambda_g \int_{\kappa^-}^{\kappa^+} \pi(y, z)\, dF^g(y). \tag{12}$$

Estimating such contributions are very useful for the purpose of policy evaluation in order to assess the sources of success or failure of poverty alleviation programs, especially with respect to the issue of the appropriate targeting of these programs.

Corollary 1.

$$C_g(\boldsymbol{y}_B, z) - C_g(\boldsymbol{y}_A, z) \leq C_g(\boldsymbol{y}_{B'}, z) - C_g(\boldsymbol{y}_{A'}, z), \quad \forall P \in \Pi_r^s, z \in [z^-, z^+], \tag{13}$$

$$\textit{iff} \quad \lambda_g^B P^j(\boldsymbol{y}_B, z) - \lambda_g^A P^j(\boldsymbol{y}_A, z) \leq \beta\big(\lambda_g^{B'} P^j(\boldsymbol{y}_{B'}, z) - \lambda_g^{A'} P^j(\boldsymbol{y}_{A'}, z)\big)$$

$$\forall z \in [z^-, z^+],\ j = r+1, \dots s \tag{14}$$

$$\textit{and} \quad \lambda_g^B P^s(\boldsymbol{y}_B, z) - \lambda_g^A P^s(\boldsymbol{y}_A, z) \leq \beta\big(\lambda_g^{B'} P^s(\boldsymbol{y}_{B'}, z) - \lambda_g^{A'} P^s(\boldsymbol{y}_{A'}, z)\big) \quad \forall z \leq z^+. \tag{15}$$

Proof. Same proof as the one for Proposition 1 provided in appendix A, but considering (12) instead of (1) as the starting point. □

Corollary 1 shows how scaling down P^j by population shares makes it possible to move from comparisons of poverty variation to the comparisons of changes in poverty contributions. In section 3.2, the usefulness of this corollary is illustrated by testing whether China's contribution to the observed decline of poverty was more important than the contribution of the other countries. It can also be used to compare changes at the national level with changes in the contribution of some specific group of the population or to provide robust bounds for the contribution of a population subgroup to national poverty.

3 HAS POVERTY BEEN HALVED DURING THE MDGS ERA?

In the present section, tools introduced previously are now applied to the analysis of poverty alleviation during the MDGs era. More specifically, we consider how true can be regarded the statement that poverty has been halved during that period, both at the aggregate level and, individually, for a substantial set of developing countries. For the two types of exercises, we are quite conservative regarding the choice of the maximum value z^+ for the poverty line and use the World Bank's current value for the extreme poverty line, that is $1,90 per day and per person in 2011 $PPP. Although we also try to consider larger values for z^+, the use of the World Bank's international poverty line means that our results mostly address the issue of robustness with respect to the choice of the poverty index. Finally, we use the $1.66 per day poverty line suggested by Klasen et al. (2015) as the value for the lower bound z^-.

3.1 DATA AND METHODOLOGY

Stochastic dominance tests typically require the use of micro-data from households surveys so as to get the individual income distributions we want to compare. Although the availability of micro-data has remarkably increased since the beginning of the XXIst century, it remains quite difficult to get the necessary data to perform the dominance tests for a large number of developing countries as well as to build consistent income distribution at the global level.

To circumvent these problems, we have to turn to secondary datasets that compile partial information from original households surveys. Here, we follow the method used in Bresson (2014) and rely on the data on mean income and the Lorenz curve given by the World Bank's poverty calculator and platform PovcalNet. Income distributions for each country and each date are then estimated using a two-step procedure. First, the income distribution is modeled parametrically using different functional forms. In the present paper, our attention is confined to the lognormal, Singh-Maddala, Dagum, and Beta 2 distributions as well as Maddala and Singh's (1977) parametric Lorenz curve. These proved to be the most interesting functional forms in Bresson (2009). The estimation showing the best fit as expressed by the sum of squared residuals is chosen for each distribution. A random sample of $10,000$ values—samples are limited to $2,000$ random values for the construction of the global distributions of income—is then generated from the parametric distribution and adjusted using Shorrocks and Wan's (2008) procedure in order to get a perfect fit with respect to the information on the Lorenz curve available from PovcalNet.

The use of PovcalNet as a source for our dataset is very interesting regarding the quality of information since the current version of PovcalNet provides information on most distribution using population centiles. This makes it possible to use generated income distribution that fit close well original income series. In the specific case of China, India, and Indonesia, representing about 40% of the World population, PovcalNet give separate information for rural and urban areas. Since average income is also available for each area, the estimations of rural and urban distributions are first treated separately in the present paper. The resulting urban and rural distributions for each country and each year are then merged and weighted using population size data from the World Bank (World Development Indicators).

An other appeal of PovcalNet is that coverage is sufficiently large both to build global income distributions over our period of analysis and to consider individual achievements for a relatively large number of developing countries. Regarding these individual achievements, we consider the case of 90 developing countries. Countries have been selected using the following criteria: *i)* at least two surveys are available, *ii)* the two surveys are spaced by at least 5 years (and up to 25 years), *iii)* at least twenty points are available for the Lorenz curve, and *iv)* the value of the headcount ratio associated with the initial distribution is larger than 1%. Although the time span is sometime relatively short, the median duration for our 90 spells is 16 years and in 23 cases it exceeds 20 years—for a list of selected countries as well as the corresponding period of analysis, see table 4. In this later case, we can be quite confident that data makes it possible to provide a consistent appraisal of the country achievements regarding poverty alleviation during the MDGs era.

Selected observations differed for the analysis of poverty changes at the global level. Indeed, we first have to consider both developed and developing countries. Secondly, we had to strike a balance between three conflicting objectives, namely ensuring a good coverage of the World Population, performing consistent comparisons (that is using the same set of countries for both the initial and final distributions and the same monetary concept), and having the longest duration. Due to poor data availability regarding income distributions in the 1990s, we had to be less ambitious regarding the duration of the period of analysis and chose observations for the periods 2000–2004, and 2010–2014, in order to get estimates of the World income distribution for the years 2002 and 2012.[13] We then obtained distributional information for 109 countries that accounted for 82.5% and 81.8% of the World population respectively in 2002 and 2012. Since PovcalNet provides for each country the mean income expressed in Purchasing Power Parity using the 2011 ICP, income values are directly comparable at the global level and over the two periods. Of course, like most studies that estimate the World distribution of income using household survey data, we could not avoid the simultaneous use of both income and consumption data, so that, as stressed by Anand and Segal (2008, p.74): "it is therefore not clear exactly what type of global distribution emerges from combining these surveys [based on either income or consumption expenditures]." This may be a serious concern

[13]An exception is Côte d'Ivoire for which data for the year 2015 were used.

for distribution changes, notably because income and consumption are imperfectly correlated. Here, we tried to fix that issue by selecting data for each country in a consistent manner so that all corresponding distributions either refer to income or consumption. So, for a given country, observed distributional changes are generally not due to changes in the underlying concept. Of course, this does not preclude other methodological changes, like a modified sampling design or a different coverage of income sources, that may affect the shape of the observed income distribution. Regarding the estimated global income distribution, the mixture of both income and consumption surveys may still be a matter of concern as the relative share of income and consumption data are not constant in our series. Indeed, these variations are prominently caused by demographic changes and there is no way of proving that results presented in the next section are not partly due to the imperfect structure of our initial dataset.

3.2 Results

We first consider changes in the World distribution of income using Proposition 1. More specifically, two claims have been tested, namely *i)* that the pace of poverty reduction over the period 2002–2012 was consistent with a 50% decrease of poverty between 1990 and 2015, and, more ambitiously *ii)* that poverty was effectively (at least) halved between 2002 and 2012. In the case of claim *i)*, the targeted change is a 24.2% decline of poverty between 2002 and 2012. Figure 1 presents results for both claims: The curves labeled "target" and "half" indicating the one that shall be compared with the 2012 cdf to assess respectively the validity of claims *i)* and *ii)*.

Before assessing the robustness of poverty changes, it is worth stressing that Figure 1a confirms the widely documented decline in global poverty during the MDGs period since the distribution for 2012 first-order dominates the distribution for 2002. Regarding the assessment of the magnitude of this lessening, the comparison of the 2012 curve with the "target" curve shows that the pace of poverty reduction between 2002 and 2012 was unambiguously larger than the one required to halve poverty over the period 1990-2015. It can also be seen that this result can easily be extended beyond the \$1.90 poverty line since no crossing can be noted at least up to \$3.00 per day.

Regarding the more challenging result of halving the share of the population living in extreme poverty between 2002 and 2012, we observe that such an important decline was reached during this subperiod since the headcount index, using the international poverty line, for 2012 was 46.8% of the estimated value for 2002.[14] Fig-

[14] Our estimates are respectively 24.9% and 11.7% for the years 2002 and 2012. The World Bank estimates provided by PovcalNet indicate that the value of the headcount ratio for 2012 was 49.9% of the value for 2002. Turning to the poverty gap index and the squared poverty gap index, this ratio becomes respectively 45.5% and 44.9%. With our own estimates, the estimated values for this ratio are respectively 42.2% and 41%.

Table 1: Upper admissible bound for the poverty domain and uncertainty intervals for global poverty reduction, 2002–2012.

Parameter	Π_0^1	Π_0^2	Π_1^2	Π_0^3	Π_1^3	Π_2^3
$z^+_{\max}$	62.3	62.3	81.5	62.3	81.5	102.1
$\beta^-(\times 100)$	37.9	39.8	39.8	40.4	40.4	40.7
$\beta^+(\times 100)$	47.9	47.3	46.4	47.3	46.1	46.1

Note: $z^+_{\max}$ denotes the minimal value for z^+ above which it cannot be concluded that poverty in 2012 is less than half its level for 2002. β^- and β^+ are estimated using \$1.66 and \$1.90 respectively for z^- and z^+.

ure 1a also bears out that this conclusion, namely that poverty has been more than halved during this period, is also valid for the whole set of additively decomposable monotone poverty indices and all poverty lines up to the internal poverty line. However, it can be seen (see Table 1) that robustness vanishes once we try to consider relatively small increments for z^+ above that threshold. Indeed, with poverty lines above \$62 per month (\$2.07 per day), the curve associated with the 2012 distribution passes above the curve corresponding to half the cdf of the 2002 distribution. Of course, previous studies have stressed a robust decline of poverty during the 1990s (see for instance Sala-i-Martin, 2006, Chen and Ravallion, 2010), so that we can be confident when claiming that a robust halving of poverty should be observed over the whole period 1990-2015 using a slightly larger interval for the poverty line.

Another way of extending the range of admissible values for the poverty line that are consistent with this robust halving of global poverty is to consider narrower classes of poverty indices. As noticed in section 2, moving to Π_0^2 is of little help since we still have to consider the results of Figure 1a over $[z^-, z^+]$ to get a robust ordering. However, turning to the class Π_1^2 of poverty indices, so that robustness can be assessed with the unique use of second order stochastic dominance conditions, proves to be useful. Unsurprisingly, Figure 1b shows a second order dominance relationship using the international poverty line as the value for z^+, but its main interest is to show that, focusing on the subset of additively decomposable monotone poverty indices that comply with the strong transfer axiom, the conclusion of a robust halving of poverty between 2002 and 2012 holds up to a maximum value of \$82 per month. Considering ordering conditions based only on third-order dominance tests, that is focusing on poverty indices from Π_2^3, the estimated value is pushed up to \$102 per month.

The last two lines of Table 1 give a complementary view on these results. It indicates the critical values β_s^- and β_s^+ that determine the range of values for β that are not associated with a crossing of the curves corresponding to $P^s(\boldsymbol{y}_{2012}, z)$ and $\beta P^s(\boldsymbol{y}_{2002}, z)$, $s \in \{1, 2, 3\}$, up to z^+ and so to an unambiguous result for the sign of the difference $P(\boldsymbol{y}_{2012}, z) - \beta P(\boldsymbol{y}_{2002}, z)$. In other words, for a given value of z^+, this can be regarded as the range of admissible values for the ratio of the poverty level in 2012 over the poverty level in 2002, using members from Π_0^1 to Π_2^3. In the theoretical case $\beta_s^- = \beta_s^+ = \beta$, final poverty would exactly be β times its initial level, whatever the chosen poverty index (within Π_s) and the value of the poverty line (up to z^+). As a consequence the difference $\beta_s^+ - \beta_s^-$ can be read as a measure of the uncertainty regarding the relative change in poverty. More generally, it can be interpreted as a measure of dissimilarity (up to a scaling factor) of the bottom part of the compared distributions.

Using the international poverty line, Table 1 shows a relatively precise view of the decline of poverty since our estimates indicate that extreme poverty at the global level in 2012 represents between 37.9% and 47.9% of its 2002 level, considering additively decomposable monotone poverty indices. With narrower classes of poverty indices, it is possible to be more precise regarding the magnitude of poverty variation during the period. For instance, with members from Π_2^3, our results show that poverty in 2012 was at best 40.7% but for sure no more than 46.1% of its 2012 level.

Raising the value for z^+ above the international poverty line logically widens these ranges of admissible values for β (see Figure 2), but is worth pointing that the effect is not symmetric. Indeed, we do not observe any effect regarding the largest magnitude of poverty alleviation (i.e. $1 - \beta^-$), whereas the lowest admissible value for that decline (i.e. $1 - \beta^+$) shrinks as z^+ increases. The picture is approximately the same considering the critical values of β associated with a second and third order dominance relationship (Figures 2b and 2c).

Before turning to individual achievements among our set of developing countries, we want to stress the influence of Chinese progress on our results. As noted for instance by Chen and Ravallion (2010), China's spectacular results with respect to poverty alleviation since the early 1980s played a crucial role in the dramatic decrease of global poverty observed during the last decades. Figure 3 shows the results of Proposition 1 using a first-order dominance test when China is dropped from the set of countries used to build the global distributions of income. We still observe a robust decline of poverty over the period 2002–2012 and the pace of poverty reduction over this period was also robustly in line with poverty halving between 1990 and 2015 (i.e. the comparison with respect to the target curve). As previously underlined, these results refer to a 10-year time span and if we consider the constant rate of return that corresponds to a 50% decrease in poverty over a 25-year period, a 24.2% decline over 10 years would have been sufficient to be on track with the MDGs poverty target. It can be seen for instance on the left panel of Figure 3 that the curve depicting the cdf for the year 2012 is everywhere above the curve associated with $(100 - 24.2)$% of the cdf for the year 2002. So whatever the poverty index from Π_0^1 and whatever the value of the poverty line—the result is valid even for high income levels—, we can be sure that the decrease in poverty was larger than 24.2% for the non-Chinese part of the World between 2002 and 2012.

So, would it have been possible to extend the duration of the spell over the whole period 1990-2015, we surely would have observed a significantly larger decrease in poverty for the non-Chinese World, hence making it possible to conclude again that the rate of poverty reduction over the studied period was consistent with halving poverty between 1990 and 2015, even when disregarding the huge Chinese contribution. However, Figure 3 and critical values for β also confirm that the decrease was not large enough to halve poverty robustly between 2002 and 2012. Indeed, the curve associated with the cdf for the year 2012 crosses the curve corresponding to 50% of the cdf of the 2002 distribution of income over the interval $[7.4, 16.2]$ and our estimates of critical values for β indicate that poverty reduction was between 39.9% and 55.7% over the period using members from Π_0^1. So, we cannot assure that the magnitude of poverty reduction during this period was at least 50%. Considering narrower set of poverty indices up to Π_2^3 does not change the picture. For instance, our estimates indicate that the decrease ranges between 45.5% and 55.1% considering members from Π_2^3.

Table 2: The Chinese contribution to global poverty, 2002–2012.

Parameter	Π_0^1	Π_0^2	Π_1^2	Π_0^3	Π_1^3	Π_2^3
......2002......						
$\beta^-(\times 100)$	0	0	0	0	0	0
$\beta^+(\times 100)$	36.3	32.9	32.9	32.9	32.9	31.1
......2012......						
$\beta^-(\times 100)$	0	0	0	0	0	0
$\beta^+(\times 100)$	12.2	12.2	9.8	12.2	9.8	7.6

Note: β^- and β^+ are estimated using \$1.66 and \$1.90 respectively for z^- and z^+.

This decisive role of China can also be stressed using Corollary 1 to assess its contribution to global poverty in 2002 and 2012. More specifically, we considered comparisons of the form $\lambda_c P(\boldsymbol{y}_c, z) \leq \beta P(\boldsymbol{y}_w, z)$ where the subscripts c and w respectively refer to China and the World. Our estimates indicate that, at the beginning of the period, China's contribution to global poverty was no more than 36.3% using indices from Π_0^1 and 31.1% with the narrower class Π_2^3 (Table 2).[15] At the end of period, the corresponding upper bounds were respectively 12.2% and 7.6%. Such a dramatic decline of this bound is indicative of the tremendous discrepancy between

[15]The reader can be logically be surprised that that our estimates systematically indicate that China's contribution to global poverty may have been zero whatever the class of poverty indices we consider. This result is not due to the uncertainty about the true value of the poverty line but is related to the fact that our framework does not rule out the use of poverty indices such that $\pi(y, z) = 0$ for $y \in [y^c, z]$ were y^c is the lowest value observed in our Chinese income distributions. Since y^c is larger than the lowest income in our global income distributions, it is simultaneously possible to have zero poverty in China and a strictly positive value for global poverty using the same index and the same poverty line.

China and the rest of the World in terms of poverty alleviation performances.

Table 3: Robust comparisons for relative poverty changes during the MDGs era for different classes of poverty indices.

Tested hypothesis	H	Π_0^1	Π_0^2	Π_1^2	Π_0^3	Π_1^3	Π_2^3
Increase	16	10	11	12	11	12	13
Decrease	74	62	65	66	65	66	67
On track	55	51	53	59	53	59	60
Not on track	35	14	15	15	15	15	15
Halving	41	37	40	49	42	51	53

Note: H is the headcount ratio estimated at z^+. Tests are performed using $1.66 and $1.90 respectively for z^- and z^+.

The overall non-ambiguous decline of poverty is a great achievement but shall not conceal the uneven individual progress of developing countries regarding the first MDGs' target. Table 3 summarizes the results of individual tests regarding poverty levels and poverty changes. The first two lines refer to the results of usual poverty orderings; the remaining three lines relate to relative poverty changes. Details, notably the estimated critical values of β for each class of poverty index up to Π_2^3, are reported in Tables 4 and 5 where countries are ordered according to the value of the ratio of the headcount index in the final year over the corresponding value for the initial year.

Our results highlight this heterogeneity of country trajectories as well as the difficulty to claim that the pace of poverty reduction was in line with a 50% decrease in 25 years in many countries during the MDGs era. Indeed, out of the 90 countries listed in the table, only 55 were on track of halving the share of their population living in extreme poverty. Out of these 55 cases, 51 had an estimated value for β^+ that was lower than the ratio that, for the considered period, would have been equivalent to halving poverty over the MDGs time-span, considering members from Π_0^1. This is for instance the case of Chile where our estimates indicate that extreme poverty in 2013 undoubtedly represented between 2% and 23% of its 1990 level. With indices that comply with the weak version of the transfer axiom but allow for a discontinuity at the poverty line (i.e. members from Π_0^2), only two countries out of the 55 aforementioned cases, namely Turkey and Honduras, could not be regarded robustly as "on track" during the studied period. Adding weak transfer sensitivity, that is considering members from Π_0^3 does not shrink further that list. However, it can be noted that Turkey falls short of being robustly "on track" in that case. Indeed, for the considered period, the target was $\beta = 0.737$ while the corresponding estimate for β^+ is 0.743. Although the evidence provided here is too limited to suggest a general law regarding changes in the headcount index and changes for large classes of poverty indices the headcount index belongs to, it is striking to note that changes in the headcount index finally proved to be a good proxy for tracking the capacity of having a pace of poverty reduction that was consistent with a 50% decrease in poverty.

Considering narrower sets of distribution-sensitive poverty indices that do not include the headcount index (i.e. $s \geq 2$ and $r \geq 1$) provides an even more welcome picture as it raises up to 60 the number of countries for which the pace of poverty reduction was robustly in line with halving poverty had the same trends been observed from 1990 to 2015. Indeed, as the headcount index does not belong to classes Π_r^s such that $r \geq 1$, it is effectively possible to consider a larger set of successful countries than the 55 whose decline in the headcount index was consistent with MDGs' target 1A. Unsurprisingly, we can remark that the gain in terms of ordering power of moving from the second to the third order of stochastic dominance is clearly lower than the increase associated with a move from the first to second order.

Within our sample of developing countries, 44 countries were successful in halving the share of the population living in extreme poverty between the dates of the surveys used for the present study. Focusing on poverty indices from Π_0^1, we observe that this result is robust for 37 out of these 44 countries. In connection with the results obtained at the global level, we can note that China and India belong to that set of very successful countries. It is worth noting that for two of the remaining countries, namely Turkey and Honduras, the critical values of β associated with Π_0^1 are such that $\beta = 1$ belongs to the corresponding interval, hence indicating that we cannot even conclude robustly that poverty has decreased during the respective periods. Turning to the class Π_1^2, the number of robust halving raises the count up to 49 countries. The additional increase is more modest, yet significant, considering indices from the class Π_2^3 with 53 countries that effectively halved poverty robustly during the studied period.

On the other end of poverty achievements, our results show that the pattern of changes in the income distribution would unambiguously not result in a 50% decrease of extreme poverty in 14 countries (15 when focusing on members of Π_2^3). For instance, the most optimistic estimate one could obtain for Togo during the period 2006–2015 using members from Π_1 is a 12.89% decrease in extreme poverty whereas a 22.08% decrease would have been needed over the same period to achieve the halving of extreme poverty in 25 years. Table 3 also shows that the share of population living in extreme poverty increased in 16 countries of the sample and, for 10 out of these 16 cases, the increase was robust at the first order. When considering dominance conditions based on second and third-order stochastic dominance tests, that number raises up to 12 and 13 countries respectively. It is however interesting to note that even among these 16 countries, we cannot reject the conclusion that poverty has been halved (using a specific poverty index and a specific value for the extreme poverty line) for some countries. Considering the whole set of additively decomposable monotone poverty indices, this situation is observed for Madagascar, Malawi, Timor-Leste, Uzbekistan, and Zambia. Moving to narrower classes of poverty indices does not wipe this indeterminacy out for these countries.

4 Concluding remarks

In their recent survey of the use of stochastic dominance tools for the analysis of poverty, García-Gómez et al. (2019, p. 1438) argued: "stochastic dominance does not allow to obtain cardinal results. That is, it tells you whether in one distribution poverty there is more or less poverty than in another, but you cannot know how much poorer is one population with respect to the other." This claim is challenged in the present paper. More specifically, we introduce stochastic dominance tools that address the issue of performing comparisons of poverty changes without relying on a specific poverty index and a specific value for the poverty line. In comparison with the traditional use of stochastic dominance techniques for poverty orderings, this newer use generally implies being able to provide robust conclusion for the sole class of additively decomposable poverty indices, hence ruling out conclusions regarding subgroup-consistent but not decomposable poverty indices. An application on the global distribution on income as well as for 90 developing countries demonstrates how powerful these tools can be, notably to confirm the robustness of results observed on the basis of the headcount index, as well as the additional information they can provide regarding the magnitude of poverty changes.

Finally, we showed that economic progress during the period 2002-2012 were associated not only with a decrease by more than 50% of the World population living in extreme poverty but also with a robust halving of the degree of poverty. Although China played a major role in this success, the good news is that the pace of poverty reduction in the rest of the World between 2002 and 2012 was consistent with a 50% decrease in poverty between 1990 and 2015, that is the reference period for Millennium Development Goals' target 1A. However, a closer inspection of individual performance with respect to poverty alleviation points out unbalanced results. Whereas some countries have succeeded in halving poverty or to have distribution change patterns that were consistent with this objective over a 25 year period, our results indicate robust observations of limited progress or poverty increases in a non-negligible number a countries, hence making the fight against poverty still a priority on the development agenda.

Appendix

A Proof of Proposition 1

Let $\tilde{P}^j_{\boldsymbol{y}}(z) := \frac{1}{(j-1)!}P^{\alpha}_{\boldsymbol{y}}(z)$. As shown by Fishburn (1976), this definition of $\tilde{P}^j_{\boldsymbol{y}}(z)$ is equivalent to the recursive definition $\tilde{P}^{j+1}_{\boldsymbol{y}}(z) = \int_{\kappa^-}^{z} \tilde{P}^j_{\boldsymbol{y}}(x)\,dx$, $j \in \mathfrak{N}\backslash\{0\}$, with $\tilde{P}^1_{\boldsymbol{y}}(z) = F(z)$.

Since it is assumed that $\pi(y,z)=0\ \forall y\in[z,\kappa^+]$, the poverty level associated with the income distribution $\boldsymbol{y}$ is:

$$P_{\boldsymbol{y}}(z)=\int_{\kappa^-}^{\kappa^+}\pi(y,z)\,dF(y)=\int_{\kappa^-}^{z}\pi(y,z)\,dF(y). \tag{16}$$

Let $\pi^{(s)}$ denote the s-th derivative of π with respect to y with $\pi^{(0)}=\pi$. We assume that there exists m points $x_1,\dots x_m$ from $]\kappa^-,z]$ where $\pi^{(s)}$ is likely to be not differentiable. So as to ease the manipulation of piecewise individual poverty functions, we note $\pi^{(s)}_{[k]}$ the k-th piece of $\pi^{(s)}$, so that $\pi^{(s)}(y,z)=\pi^{(s)}_{[k]}(y,z)$ for $y\in[x_{k-1},x_k[$, $k\in\{1,\dots m\}$. Moreover, we assume that $\pi^{(s)}_{[k]}(x_k,z)$ exists, has a finite value, and shall be understood as $\lim_{x\uparrow x_k}\pi^{(s)}_{[k]}(x,z)$. Finally, let $x_0=\kappa^-$ and $x_m=z$. We then have:

$$P_{\boldsymbol{y}}(z)=\sum_{k=1}^{m}\int_{x_{k-1}}^{x_k}\pi_{[k]}(y,z)\,dF(y). \tag{17}$$

A.1 Sufficiency part

Using integration by parts on the right-hand term of (17), we obtain:

$$P_{\boldsymbol{y}}(z)=\sum_{k=1}^{m}\left(\left[\pi_{[k]}(y,z)\tilde{P}^1_{\boldsymbol{y}}(y)\right]_{x_{k-1}}^{x_k}-\int_{x_{k-1}}^{x_k}\pi^{(1)}_{[k]}(y,z)\tilde{P}^1_{\boldsymbol{y}}(y)\,dy\right), \tag{18}$$

$$=\pi_{[m]}(z,z)\tilde{P}^1_{\boldsymbol{y}}(z)+\sum_{k=1}^{m-1}\left(\pi_{[k]}(x_k,z)-\pi_{[k+1]}(x_k,z)\right)\tilde{P}^1_{\boldsymbol{y}}(x_k)$$
$$-\sum_{k=1}^{m}\int_{x_{k-1}}^{x_k}\pi^{(1)}_{[k]}(y,z)\tilde{P}^1_{\boldsymbol{y}}(y)\,dy, \tag{19}$$

since, by definition, $\tilde{P}^1_{\boldsymbol{y}}(\kappa^-)=0$.

Now, we consider the hypothesis that, for some $s\in\mathfrak{N}\setminus\{0\}$, we have:

$$P_{\boldsymbol{y}}(z)=\sum_{j=1}^{s}(-1)^{j-1}\pi^{(j-1)}_{[m]}(z,z)\tilde{P}^j_{\boldsymbol{y}}(z)+(-1)^{s-1}\sum_{k=1}^{m-1}\left(\pi^{(s-1)}_{[k]}(x_k,z)-\pi^{(s-1)}_{[k+1]}(x_k,z)\right)\tilde{P}^s_{\boldsymbol{y}}(x_k)$$
$$+(-1)^s\sum_{k=1}^{m}\int_{x_{k-1}}^{x_k}\pi^{(s)}_{[k]}(y,z)\tilde{P}^s_{\boldsymbol{y}}(y)\,dy, \tag{20}$$

With the assumption that $\pi^{(s-1)}_{[k]}$ is continuous over $[\kappa^-,z[$, so that $\pi^{(s-1)}_{[k]}(x_k,z)=\pi^{(s-1)}_{[k+1]}(x_k,z)$, $\forall k\in\{1,\dots m-1\}$, the second element of the right-hand term in (20) vanishes. Integrating by parts the last element of the right-hand term in (20), we obtain:

$$P_{\boldsymbol{y}}(z)=\sum_{j=1}^{s}(-1)^{j-1}\pi^{(j-1)}_{[m]}(z,z)\tilde{P}^j_{\boldsymbol{y}}(z)+(-1)^s\sum_{k=1}^{m}\left[\pi^{(s)}_{[k]}(y,z)\tilde{P}^{s+1}_{\boldsymbol{y}}(y)\right]_{x_{k-1}}^{x_k}$$

$$- (-1)^s \sum_{k=1}^{m} \int_{x_{k-1}}^{x_k} \pi_{[k]}^{(s+1)}(y,z) \tilde{P}_{\boldsymbol{y}}^{s+1}(y)\, dy, \tag{21}$$

$$= \sum_{j=1}^{s+1} (-1)^{j-1} \pi_{[m]}^{(j-1)}(z,z) \tilde{P}_{\boldsymbol{y}}^{j}(z) + (-1)^s \sum_{k=1}^{m-1} \left(\pi_{[k]}^{(s)}(x_k,z) - \pi_{[k+1]}^{(s)}(x_k,z) \right) \tilde{P}_{\boldsymbol{y}}^{s+1}(x_k)$$
$$+ (-1)^{s+1} \sum_{k=1}^{m} \int_{x_{k-1}}^{x_k} \pi_{[k]}^{(s+1)}(y,z) \tilde{P}_{\boldsymbol{y}}^{s+1}(y)\, dy, \tag{22}$$

since $P_{\boldsymbol{y}}^{s+1}(\kappa^-) = 0$. It can be easily checked that both (20) and (22) are consistent with (19) with the appropriate choice for s. As (22) is obtained from (20), we can recursively see that (22) is true $\forall j \in \{1, \dots, s\}$. In the case $\pi_{[m]}^{(j)}(z,z) = 0$ $\forall j \in \{0, \dots r-1\}$, (20) simplifies to:

$$P_{\boldsymbol{y}}(z) = \sum_{j=r}^{s} (-1)^{j-1} \pi_{[m]}^{(j-1)}(z,z) \tilde{P}_{\boldsymbol{y}}^{j}(z) + (-1)^{s-1} \sum_{k=1}^{m-1} \left(\pi_{[k]}^{(s-1)}(x_k,z) - \pi_{[k+1]}^{(s-1)}(x_k,z) \right) \tilde{P}_{\boldsymbol{y}}^{s}(x_k)$$
$$+ (-1)^s \sum_{k=1}^{m} \int_{x_{k-1}}^{x_k} \pi_{[k]}^{(s)}(y,z) \tilde{P}_{\boldsymbol{y}}^{s}(y)\, dy. \tag{23}$$

Now, let's consider the comparison of poverty variations. Let $D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) := P_{\boldsymbol{x}}(z) - P_{\boldsymbol{y}}(z) - \beta\big(P_{\boldsymbol{x}'}(z) - P_{\boldsymbol{y}'}(z)\big)$ and $D^j(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) := \frac{1}{(j-1)!}\Big(P_{\boldsymbol{x}}^j(z) - P_{\boldsymbol{y}}^j(z) - \beta\big(P_{\boldsymbol{x}'}^j(z) - P_{\boldsymbol{y}'}^j(z)\big)\Big)$. Equation (23) then makes it possible to express D_P as:

$$D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) = \sum_{j=r}^{s} (-1)^{j-1} \pi_{[m]}^{(j-1)}(z,z) D^j(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta)$$
$$+ (-1)^{s-1} \sum_{k=1}^{m-1} \left(\pi_{[k]}^{(s-1)}(x_k,z) - \pi_{[k+1]}^{(s-1)}(x_k,z) \right) D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta)$$
$$+ (-1)^s \sum_{k=1}^{m} \int_{x_{k-1}}^{x_k} \pi_{[k]}^{(s)}(y,z) D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', y, \beta)\, dy. \tag{24}$$

Having $D^j(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) \leq 0$, $\forall j \in \{r, \dots s\}$, and $D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', y, \beta) \leq 0$ $\forall y \in [\kappa^-, z]$ are thus sufficient conditions for $D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) \leq 0$. As the multiplicative term $\frac{1}{(j-1)!}$ is strictly positive in $D^j(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta)$, one can equivalently focus on the sign of $P_{\boldsymbol{x}}^j(z) - P_{\boldsymbol{y}}^j(z) - \beta\big(P_{\boldsymbol{x}'}^j(z) - P_{\boldsymbol{y}'}^j(z)\big)$.

In the case π shows no discontinuity over $[\kappa^-, z[$, (24) becomes:

$$D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) = \sum_{j=r}^{s} (-1)^{j-1} \pi^{(j-1)}(z,z) D^j(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta)$$
$$+ (-1)^s \int_{x_{k-1}}^{x_k} \pi^{(s)}(y,z) D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', y, \beta)\, dy, \tag{25}$$

where $\pi^{(j)}(z,z)$ shall be interpreted as $\lim_{x \uparrow z} \pi^{(j)}(x,z)$. A quick examination shows that the satisfaction of the same conditions as above are sufficient conditions for $D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) \leq 0$.

A.2 Necessity part

To prove necessity, we have to assume that either $\tilde{P}^j_{\boldsymbol{x}}(z) - \tilde{P}^j_{\boldsymbol{y}}(z) > \beta\big(\tilde{P}^j_{\boldsymbol{x}'}(z) - \tilde{P}^j_{\boldsymbol{y}'}(z)\big)$, $\forall j \in \{r, \ldots s\}$, or that there exists $\bar{y} \in]\kappa^-, z[$ such that $\tilde{P}^s_{\boldsymbol{x}}(y) - \tilde{P}^s_{\boldsymbol{y}}(y) > \beta\big(\tilde{P}^s_{\boldsymbol{x}'}(y) - \tilde{P}^s_{\boldsymbol{y}'}(y)\big)$. For the first set of conditions, we consider individual poverty indices $\bar{\pi}_t$ that generalize the one used in Zheng (1999, proof of Proposition 3) and defined by:

$$\bar{\pi}_t(y, z) := \begin{cases} \frac{1}{t!}(z-y)^t + \sum_{j=0}^{t-1} \gamma_j (z-y)^j & \text{if } y < z \\ 0 & \text{otherwise} \end{cases}, \tag{26}$$

with $\gamma_j > 0 \; \forall j \in \{r, \ldots t-1\}$. It can be checked that poverty indices of the form (1) with $\pi = \bar{\pi}_t$ belong to Π^s_r as long as $t \geq s$. Assuming $t = s$, equation (24) then becomes:

$$\begin{aligned} D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) = D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) + \sum_{j=r}^{s-1} \gamma_j D^j(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) \\ + \int_{x_{k-1}}^{x_k} D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', y, \beta)\, dy. \end{aligned} \tag{27}$$

Let us assume that $D^j(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) \leq 0$, for $j \neq j'$, and $D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', y, \beta) \leq 0$ $\forall y \in [\kappa^-, z]$. Then, if $D^{j'}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) \leq 0$, we have $D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) \leq 0$. On the other hand, if $D^{j'}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) > 0$, there exists a sufficiently large value $\gamma_{j'}$ such that $D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) > 0$. This proves the necessity of $D^j(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) \leq 0$, $\forall j \in \{r, \ldots s\}$.

Now, let's turn to the necessity of $D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', y, \beta) \leq 0 \; \forall y \in [\kappa^-, z]$. We consider the family of individual poverty measures $\hat{\pi}_t$ implicitly used in Duclos and Makdissi (2004, proof of Proposition 1) and such that $\forall t \in \mathfrak{N} \setminus \{0\}$:

$$\hat{\pi}_t(y, z) := \begin{cases} \sum_{j=1}^{t} (-1)^{j-1} \frac{\varepsilon^j}{j!(t-j)!} (\hat{y} + \varepsilon - y)^{t-j} & \text{if } y \leq \hat{y} \\ \frac{1}{t!}(\hat{y} + \varepsilon - y)^t & \text{if } \hat{y} < y \leq \hat{y} + \varepsilon \\ 0 & \text{otherwise} \end{cases}, \tag{28}$$

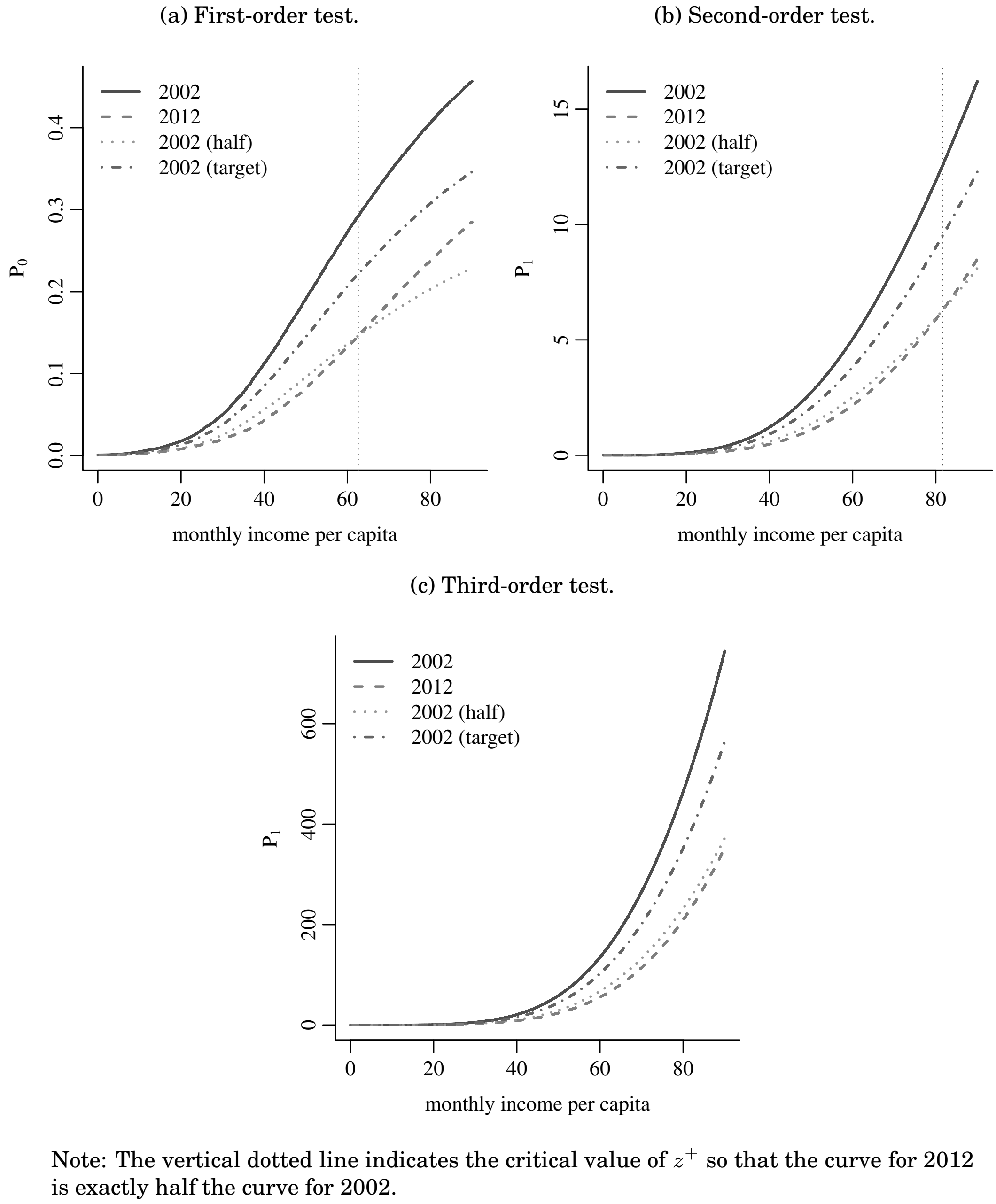
with ε strictly positive, arbitrarily close to 0, and $\hat{y} + \varepsilon < z$. It can be checked that poverty indices of the form (1) with $\pi = \hat{\pi}_t$ belong to Π^s_r as long as $t \geq s$. Whatever $t \in \mathfrak{N} \setminus \{0\}$, it can also be shown that the t-th derivative of $\hat{\pi}_t(y, z)$ with respect to individual income is:

$$\hat{\pi}_t^{(t)}(y,z) := \begin{cases} 0 & \text{if } y \leq \hat{y} \\ (-1)^t & \text{if } \hat{y} < y \leq \hat{y} + \varepsilon \\ 0 & \text{otherwise} \end{cases} . \tag{29}$$

Moreover, $\hat{\pi}_t^{(j)}(z,z) = 0\ \forall j \in \{1, \dots s\}$ and $\hat{\pi}_t^{(s)}$ is continuous over $\boldsymbol{K}$. Considering the case $t = s$, equation (24) thus becomes:

$$D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) = \int_{\hat{y}}^{\hat{y}+\varepsilon} D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', y, \beta)\, dy. \tag{30}$$

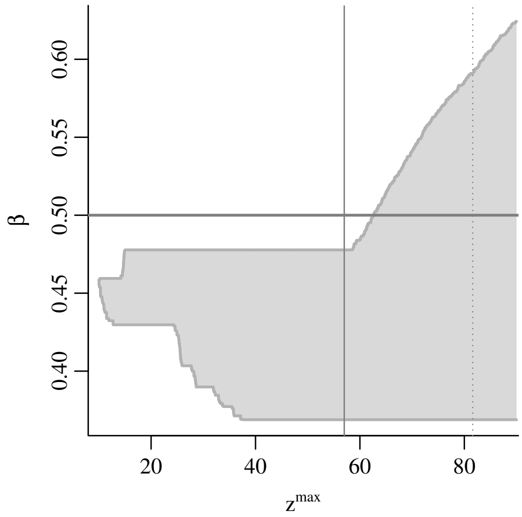
Let's assume that $D^j(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) = 0\ \forall j \in \{r, \dots s\}$. If $D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', y, \beta) > 0$ for $y \in [\hat{y}, \hat{y} + \varepsilon] \subset [\kappa^-, z]$, then $D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) > 0$. On the contrary, if $D^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', y, \beta) \leq 0\ \forall y \in [\kappa^-, z]$, then $D_P(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}', z, \beta) \leq 0$. As this is true $\forall \hat{y} \in [\kappa^-, z[$, this proves the necessity of the condition.



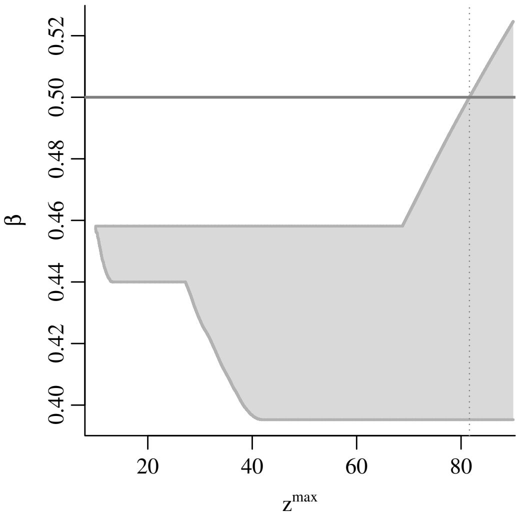
Note: The vertical dotted line indicates the critical value of z^+ so that the curve for 2012 is exactly half the curve for 2002.

Figure 1: Poverty reduction in the World, 2002–2012: dominance tests.

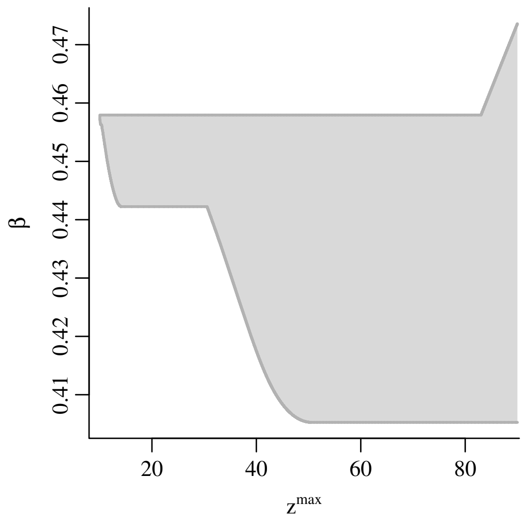
(a) First-order test.



(b) Second-order test.



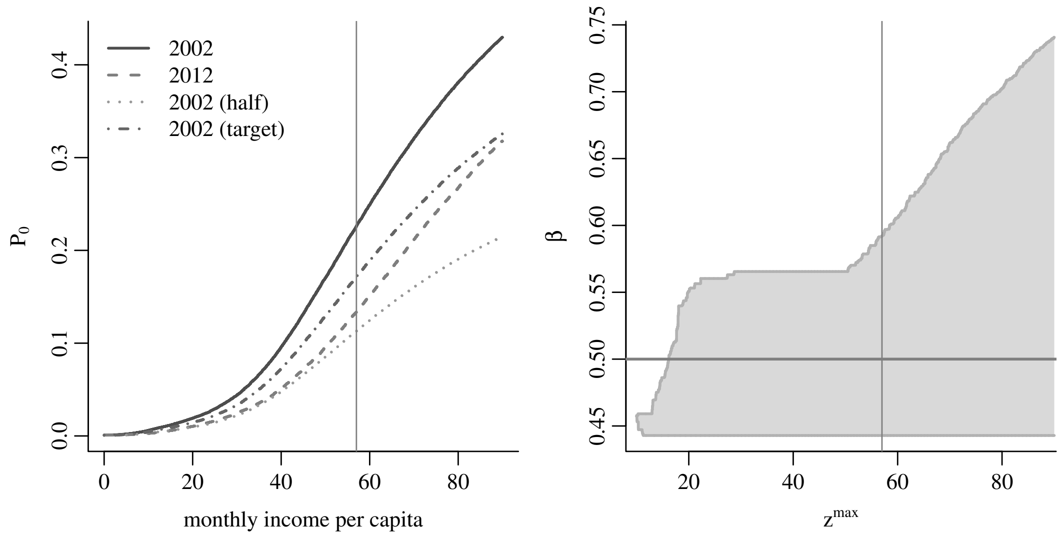
(c) Third-order test.



Note: The vertical dotted line indicates the critical value of z^+ so that the curve for 2012 is exactly half the curve for 2002.

Figure 2: Uncertainty interval for different values of z^+, 2002–2012.

(a) First-order test. (b) Uncertainty interval ($s = 1$).



Note: The vertical thin line corresponds to the monthly income associated with the $1.90 poverty line.

Figure 3: Poverty reduction in the World, 2002–2012: dominance tests (omitting China).

B Additional tables

Table 4: Confidence bounds for poverty alleviation in 90 countries, 1990-2015, part 1.

				Π_0^1		Π_0^2	
Country	Period	Target (%)	Ratio (%)	$\beta^-{\scriptstyle(\times 100)}$	$\beta^+{\scriptstyle(\times 100)}$	$\beta^-{\scriptstyle(\times 100)}$	$\beta^+{\scriptstyle(\times 100)}$
Thailand	1990–2013	52.85	0.3337	0.2016	1.471	0.2016	0.7716
Mongolia	2002–2014	71.7	2.161	0.3584	2.187	0.3584	2.187
China	1990–2013	52.85	4.092	0.5112	5.939	0.8694	3.645
Kyrgyz Rep.	1998–2014	64.17	4.158	0.1359	4.177	0.1359	4.177
Algeria	1988–2011	52.85	4.786	1.087	4.921	1.087	4.921
Bhutan	2003–2012	77.92	5.994	0.495	6.051	0.495	6.051
Vietnam	1992–2014	54.34	6.148	1.02	6.16	1.02	6.16
Cambodia	1994–2012	60.71	6.618	0.3676	6.628	0.3676	6.628
Azerbaijan	1995–2008	69.74	6.648	1.493	6.831	1.493	6.831
Pakistan	1990–2013	52.85	9.766	0.1309	9.771	0.1309	9.771
Armenia	1999–2014	65.98	13.01	1.887	13.81	1.887	13.81
Chile	1990–2013	52.85	13.22	1.98	22.99	1.98	17.66
El Salvador	1991–2014	52.85	14	0.1825	14.01	0.1825	14.01
Indonesia	1993–2014	55.86	14.16	0.113	13.86	0.113	13.86
Costa Rica	1990–2014	51.41	16.53	0.3472	17.06	0.3472	16.55
Nicaragua	1993–2014	55.86	16.61	0.2545	16.62	0.2545	16.62
Panama	1991–2014	52.85	17.26	0.3322	17.26	0.3322	17.26
Brazil	1990–2014	51.41	17.81	1.389	65.67	1.389	47.65
Egypt	1990–2015	50	17.94	4	18.32	4	18.32
Turkey	2002–2013	73.71	19.53	19.16	200	19.16	83.4
Peru	1994–2014	57.43	19.56	9.524	25	9.628	19.79
Sri Lanka	1990–2012	54.34	20.92	6.25	50	14.2	50
Paraguay	1995–2014	59.05	21.27	1.527	21.3	1.527	21.3
Ecuador	1994–2014	57.43	22.08	1	24.34	1	22.41
Nepal	1995–2010	65.98	23.58	1.351	23.58	1.351	23.58
Fiji	2002–2013	73.71	25.33	8.333	26.18	12.94	26.18
Ghana	1991–2012	55.86	28.35	4.762	28.36	4.762	28.36
Mauritania	1995–2014	59.05	28.4	5.556	29.15	5.556	29.15
Mexico	1992–2014	54.34	31.29	16.67	57.14	23.78	51.68
Tajikistan	1999–2014	65.98	35.2	0.9259	35.2	0.9259	35.2
Bolivia	1997–2014	62.42	35.49	1.786	35.55	1.786	35.55
Jamaica	1990–2004	67.83	37.62	7.143	54.67	7.143	44.93
Dominican Rep.	1996–2015	59.05	37.94	0.9901	37.94	0.9901	37.94
Bangladesh	1991–2010	59.05	41.33	3.571	41.39	3.571	41.39
Tonga	2001–2009	80.11	42.86	11.11	44.49	11.11	44.49
Philippines	1997–2015	60.71	46.17	5.882	46.2	5.882	46.2
Morocco	1998–2006	80.11	46.19	6.667	46.5	6.667	46.5
India	1993–2011	60.71	46.43	7.199	45.19	7.199	45.19
Honduras	1991–2014	52.85	47.37	40.62	2400	44.17	2400
South Africa	1996–2011	65.98	48.26	16.67	60	16.67	48.35
Ethiopia	1995–2010	65.98	49.43	5.263	49.43	5.263	49.43
Cabo Verde	2001–2007	84.67	49.49	5	49.51	5	49.51

				Π_0^1		Π_0^2	
Country	Period	Target (%)	Ratio (%)	β^- (×100)	β^+ (×100)	β^- (×100)	β^+ (×100)
Cameroon	1996–2014	60.71	50.19	46.52	1000	48.11	1000
Uganda	1992–2012	57.43	50.26	1.639	50.26	1.639	50.26
Burkina Faso	1994–2014	57.43	51.1	0.0721	51.1	0.0721	51.1
Solomon Islands	2005–2013	80.11	54.45	1.266	54.45	1.266	54.45
Niger	1994–2014	57.43	54.73	0.3226	54.73	0.3226	54.73
Georgia	1997–2014	62.42	55.44	1.538	55.5	1.538	55.5
Liberia	2007–2014	82.36	56.07	1.351	56.08	1.351	56.08
Mali	1994–2009	65.98	57.02	0.1033	57.02	0.1033	57.02
Chad	2003–2011	80.11	60.19	34.83	300	44.14	300
Albania	1997–2012	65.98	63.58	27.27	100	27.27	81.11
Gambia	1998–2003	87.06	64.5	1.852	64.5	1.852	64.5
Tanzania	1991–2011	57.43	65.63	1.493	65.64	1.493	65.64
Tunisia	2005–2010	87.06	65.67	18.18	71.61	32	68.59
Lao	1992–2012	57.43	67.25	31.65	200	45.96	200
Maldives	2002–2009	82.36	69.94	6.897	70.01	6.897	70.01
Guatemala	1998–2014	64.17	70.19	14.29	70.23	14.29	70.23
Namibia	2003–2009	84.67	71.01	23.68	71.1	23.68	71.06
Guinea	1994–2012	60.71	71.56	23.43	266.7	31.08	266.7
Papua N. Guinea	1996–2009	69.74	72.57	0.3442	72.57	0.3442	72.57
Congo, Rep.	2005–2011	84.67	73.79	71.45	1100	73.28	1100
Senegal	2001–2011	75.79	77.69	70.18	3000	76.18	3000
Rwanda	2000–2013	69.74	77.7	1.064	77.71	1.064	77.71
Mozambique	1996–2008	71.7	80.02	37.64	700	44.54	598.7
Congo, Dem. Rep.	2004–2012	80.11	81.51	5.051	81.51	5.051	81.51
Lesotho	1994–2010	64.17	85.64	6.25	85.8	12.68	85.8
Swaziland	2000–2009	77.92	86.68	86.64	1467	86.64	671.3
Argentina	1992–2014	54.34	87.44	2	95.28	2	89.01
Burundi	1998–2013	65.98	87.51	0.4545	87.53	0.4545	87.53
Togo	2006–2015	77.92	88.22	87.19	1.01e+04	87.19	1.01e+04
Sierra Leone	2003–2011	80.11	89.63	24	100	37.35	89.63
Nigeria	1992–2009	62.42	93.68	3.125	93.75	3.125	93.75
Haiti	2001–2012	73.71	97.13	96.87	5000	96.87	3057
Benin	2003–2015	71.7	101.8	101.8	2.52e+04	101.8	2.52e+04
Central Afr. Rep.	2003–2008	87.06	102.2	102.1	1.15e+04	102.1	1.15e+04
Guinea-Bissau	1993–2010	62.42	105.1	102.2	6600	105	6600
Timor-Leste	2001–2007	84.67	105.2	20	105.4	32.17	105.4
Zambia	1991–2015	51.41	105.2	0.2439	105.3	0.2439	105.3
S. Tome & Principe	2000–2010	75.79	109.8	94.65	600	108.9	467
Djibouti	2002–2013	73.71	111	107.6	2900	107.6	2900
Malawi	1997–2010	69.74	112	40	313	40	282.5
Madagascar	1993–2012	59.05	112.5	20	221.7	20	161.2
Côte d'Ivoire	1992–2015	52.85	126.3	126.2	600	126.2	600
Comoros	2004–2013	77.92	134.3	134.2	1000	134.2	980.3
Uzbekistan	1998–2003	87.06	147.1	0.6579	151.6	0.6579	151.6
Micronesia	2005–2013	80.11	154.7	154.5	1.08e+04	154.5	1.08e+04
Kenya	1994–2005	73.71	178.9	130	900	178.1	900
Venezuela	1992–2006	67.83	216.6	216.6	277.6	216.6	277.6

Country	Period	Target (%)	Ratio (%)	Π_0^1		Π_0^2	
				$\beta^-_{(\times 100)}$	$\beta^+_{(\times 100)}$	$\beta^-_{(\times 100)}$	$\beta^+_{(\times 100)}$
Yemen	1998–2014	64.17	273.2	166.7	500	238	364

Note: Target indicates for each country the ratio of the final level of poverty over its initial level one should have observed over the studied period in order to have poverty been shrunk by 50% had the pace of poverty reduction been the same over 25 years. Ratio is the ratio of the final over the initial value of the headcount index using the $1.90 poverty line.

Table 5: Confidence bounds for poverty alleviation in 90 countries, 1990-2015, part 2.

	Π_1^2		Π_0^3		Π_1^3		Π_2^3	
Country	$\beta^-(\times 100)$	$\beta^+(\times 100)$	$\beta^-(\times 100)$	$\beta^+(\times 100)$	$\beta^-(\times 100)$	$\beta^+(\times 100)$	$\beta^-(\times 100)$	$\beta^+(\times 100)$
Thailand	0.2016	0.7716	0.2016	0.661	0.2016	0.661	0.2016	0.661
Mongolia	0.3584	1.344	0.3584	2.187	0.3584	1.344	0.3584	0.8367
China	0.8694	3.645	0.8694	3.004	0.8694	2.885	0.8694	2.885
Kyrgyz Rep.	0.1359	1.889	0.1359	4.177	0.1359	1.889	0.1359	0.9272
Algeria	1.087	3.35	1.087	4.921	1.087	3.35	1.087	2.421
Bhutan	0.495	3.555	0.495	6.051	0.495	3.555	0.495	2.378
Vietnam	1.02	4.075	1.02	6.16	1.02	4.075	1.02	3.08
Cambodia	0.3676	4.212	0.3676	6.628	0.3676	4.212	0.3676	2.733
Azerbaijan	1.493	5.333	1.493	6.831	1.493	5.333	1.493	4.625
Pakistan	0.1309	4.14	0.1309	9.771	0.1309	4.14	0.1309	2.144
Armenia	1.887	9.698	1.887	13.81	1.887	9.698	1.887	7.142
Chile	1.98	17.27	1.98	17.66	1.98	17.27	1.98	14.71
El Salvador	0.1825	6.004	0.1825	14.01	0.1825	6.004	0.1825	2.995
Indonesia	0.113	7.032	0.113	13.86	0.113	7.032	0.113	4.283
Costa Rica	0.3472	12.93	0.3472	16.55	0.3472	12.93	0.3472	9.883
Nicaragua	0.2545	8.37	0.2545	16.62	0.2545	8.37	0.2545	5.157
Panama	0.3322	8.456	0.3322	17.26	0.3322	8.456	0.3322	5.081
Brazil	1.389	47.65	1.389	42.78	1.389	42.78	1.389	42.78
Egypt	4	11.88	4	18.32	4	11.88	4	9.437
Turkey	19.16	83.4	19.16	74.32	19.16	74.32	19.16	74.32
Peru	9.628	17.26	9.628	19.79	9.628	17.26	9.628	15.92
Sri Lanka	14.2	50	17.51	45.97	17.51	45.97	17.98	45.97
Paraguay	1.527	16.03	1.527	21.3	1.527	16.03	1.527	12.64
Ecuador	1	18.56	1	22.41	1	18.56	1	17.39
Nepal	1.351	14	1.351	23.58	1.351	14	1.351	10.06
Fiji	12.94	22.55	12.94	26.18	12.94	20.68	12.94	20.45
Ghana	4.762	24.01	4.762	28.36	4.762	24.01	4.762	22.41
Mauritania	5.556	22.61	5.556	29.15	5.556	22.61	5.556	17.85
Mexico	23.78	51.68	24.76	49.11	24.76	49.11	24.76	49.11
Tajikistan	0.9259	22.66	0.9259	35.2	0.9259	22.66	0.9259	15.78
Bolivia	1.786	32.22	1.786	35.55	1.786	32.22	1.786	29.82
Jamaica	7.143	44.24	7.143	44.93	7.143	44.24	7.143	41.77
Dominican Rep.	0.9901	25.07	0.9901	37.94	0.9901	25.07	0.9901	15.98
Bangladesh	3.571	28.54	3.571	41.39	3.571	28.54	3.571	21.55
Tonga	11.11	37.25	11.11	44.49	11.11	37.25	11.11	33.73
Philippines	5.882	37.88	5.882	46.2	5.882	37.88	5.882	32.07
Morocco	6.667	40.87	6.667	46.5	6.667	40.87	6.667	35.68
India	7.199	34.84	7.199	45.19	7.199	34.84	7.199	28.56
Honduras	44.17	2400	44.18	2400	44.18	2400	46.17	2400
South Africa	16.67	40.75	16.67	48.35	16.67	37.72	16.67	35.8
Ethiopia	5.263	33.96	5.263	49.43	5.263	33.96	5.263	27.78
Cabo Verde	5	40.52	5	49.51	5	40.52	5	35.46
Cameroon	50.01	1000	48.11	1000	50.01	1000	51.46	1000
Uganda	1.639	35.49	1.639	50.26	1.639	35.49	1.639	28.28

Country	Π_1^2 $\beta^-(\times 100)$	Π_1^2 $\beta^+(\times 100)$	Π_0^3 $\beta^-(\times 100)$	Π_0^3 $\beta^+(\times 100)$	Π_1^3 $\beta^-(\times 100)$	Π_1^3 $\beta^+(\times 100)$	Π_2^3 $\beta^-(\times 100)$	Π_2^3 $\beta^+(\times 100)$
Burkina Faso	0.0721	22.16	0.0721	51.1	0.0721	22.16	0.0721	11.8
Solomon Islands	1.266	38.74	1.266	54.45	1.266	38.74	1.266	30.48
Niger	0.3226	31.11	0.3226	54.73	0.3226	31.11	0.3226	20.73
Georgia	1.538	46.36	1.538	55.5	1.538	46.36	1.538	38.5
Liberia	1.351	40.94	1.351	56.08	1.351	40.94	1.351	31.5
Mali	0.1033	28.63	0.1033	57.02	0.1033	28.63	0.1033	17.31
Chad	44.14	300	48.27	300	48.27	300	48.27	300
Albania	27.27	70.97	27.27	81.11	27.27	70.97	27.27	66.64
Gambia	1.852	48.83	1.852	64.5	1.852	48.83	1.852	40.45
Tanzania	1.493	49.02	1.493	65.64	1.493	49.02	1.493	38.86
Tunisia	32	62.04	32	68.59	32	62.04	32	56.71
Lao	45.96	200	51.24	200	51.24	200	51.24	200
Maldives	6.897	57.58	6.897	70.01	6.897	57.58	6.897	50.57
Guatemala	14.29	52.14	14.29	70.23	14.29	52.14	14.29	42.61
Namibia	23.68	65.05	23.68	71.06	23.68	65.05	23.68	59.92
Guinea	31.08	266.7	32.47	266.7	32.47	266.7	32.47	266.7
Papua N. Guinea	0.3442	56.2	0.3442	72.57	0.3442	56.2	0.3442	44.95
Congo, Rep.	74.56	1100	73.28	1100	74.56	1100	76.1	1100
Senegal	77.53	3000	76.18	3000	77.53	3000	81.52	3000
Rwanda	1.064	62.01	1.064	77.71	1.064	62.01	1.064	52.52
Mozambique	44.54	598.7	46.65	598.7	46.65	598.7	46.65	598.7
Congo, Dem. Rep.	5.051	61.18	5.051	81.51	5.051	61.18	5.051	50.66
Lesotho	12.68	85.8	13.96	85.8	13.96	85.8	13.96	85.8
Swaziland	86.64	671.3	86.64	544.9	86.64	544.9	86.64	544.9
Argentina	2	81.55	2	89.01	2	81.55	2	76.28
Burundi	0.4545	71.98	0.4545	87.53	0.4545	71.98	0.4545	61.41
Togo	94.77	1.01e+04	87.19	1.01e+04	94.77	1.01e+04	104.1	1.01e+04
Sierra Leone	37.35	76.41	41.15	89.63	41.15	76.41	41.15	67.8
Nigeria	3.125	79.09	3.125	93.75	3.125	79.09	3.125	68.26
Haiti	103.5	3057	96.87	2805	103.5	2805	107.2	2805
Benin	139.2	2.52e+04	101.8	2.52e+04	139.2	2.52e+04	201.5	2.52e+04
Central African Rep.	109.4	1.15e+04	102.1	1.15e+04	109.4	1.15e+04	117.6	1.15e+04
Guinea-Bissau	107.7	6600	105	6600	107.7	6600	111.5	6600
Timor-Leste	32.17	100.8	34.21	105.4	34.21	100.8	34.21	100.8
Zambia	0.2439	84.53	0.2439	105.3	0.2439	84.53	0.2439	69.19
S. Tome & Principe	109.7	467	108.9	467	110	467	111.5	467
Djibouti	125.1	2900	107.6	2900	125.1	2900	149.2	2900
Malawi	40	282.5	40	272	40	272	40	272
Madagascar	20	161.2	20	160	20	160	20	160
Côte d'Ivoire	136.6	600	126.2	600	136.6	600	146.7	600
Comoros	174.1	980.3	134.2	980.3	174.1	980.3	212.6	980.3
Uzbekistan	0.6579	125.5	0.6579	151.6	0.6579	125.5	0.6579	99.96
Micronesia	245	1.08e+04	154.5	1.08e+04	245	1.08e+04	406.2	1.08e+04
Kenya	183.5	900	178.1	900	183.5	900	186.8	900
Venezuela	244.1	277.6	216.6	277.6	244.1	277.6	256.3	277.6
Yemen	238	364	271	364	279.9	364	279.9	364

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