Bounding Surface Modeling of Compacted Silty Sand Exhibiting Suction Dependent Post-peak Strain Softening

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Ujwalkumar D. Patil[1], Laureano R. Hoyos[2], Mathilde Morvan[3] and Anand J. Puppala[4]

SUMMARY

This article focuses on modeling the strain hardening-softening response of statically compacted silty sand as observed from a comprehensive series of suction-controlled, consolidated-drained triaxial tests accomplished in a fully-automated, double-walled triaxial test system via the axis-translation technique. The constitutive model used in this work is based on the theory of Bounding Surface (BS) plasticity, and is formulated within a critical state framework. The essential BS model parameters are calibrated using the full set of triaxial test results and then used for predictions of compacted silty sand response at matric suction states varying from 50 to 750 kPa. Complementary simulations using the Barcelona Basic Model (BBM) have also been included, alongside BS model predictions, in order to get further enlightening insights into some of the main limitations and challenges facing both frameworks within the context of the experimental evidence resulting from the present research effort. In general, irrespective of the value of matric suction applied, the BBM performs relatively well in predicting response at peak

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and critical state failure under low net confining pressure while the BSM performs relatively well under high net confining pressures.

Keywords: unsaturated soil, matric suction, triaxial testing, dilatancy, strain-softening, elastoplasticity

1. BACKGROUND AND SCOPE

Over-consolidated and compacted silty sand type soil upon shearing are most likely to manifest the strain-induced post-peak softening along with the suction-induced dilatational volume change. Strain softening in saturated soil during shearing has been attributed to either damage to the soil structure, i.e. localization, or to the dilation, i.e. increase in volume [1]. However, hydro-mechanical response of unsaturated soil becomes complicated due to multi-phase interaction between air-solids, water-solids and air-water interface within void space of the soil skeleton. The variation of pressure difference across air-water interface at grain point contact varies with degree of saturation and contributes largely to hydro-mechanical response of soil, provided the water phase is continuous.

However, at high suction, i.e. beyond residual suction, the water phase is no longer continuous and hence the same cannot hold true. Key data set on stress-strain response, especially the one obtained via suction-controlled triaxial testing, not only plays a crucial role in accurate assessments of unsaturated soil strength-deformation but is also essential in subsequent validation of predictive models. The experimentally validated model could further be incorporated within finite element codes to study complex soil-structure interaction of man-made structures.

Alonso *et al*. [2] extended the classic Cam-Clay model to unsaturated soils based on critical state framework and could explain the wetting induced collapse phenomenon along with being able to reasonably simulate the compressive volume change and strain-hardening type triaxial response. Since then, despite efforts by many researchers in fine tuning and improving this seminal model, none of them have been able to successfully reproduce the strain-induced softening and the stress-induced dilatancy observed in heavily overconsolidated soils when compacted and tested under unsaturated state.

Bounding Surface (BS) plasticity theory was initially developed to simulate plasticity in metals [3] and was soon extended to different geomaterials such as cohesive soils [4–6]; sands [7, 8]; pavement base materials [9]; geosynthetic reinforced material [10, 11]; concrete [12]; and rockfill materials [13, 14]. The BS theory underwent further simplification and refinement by several researchers [5–7, 15–19].

Bardet [7] introduced a comprehensive BS framework for soils, including nine material constants, and was quite successful in reproducing the complex post-peak strain softening and dilatational response of dry and saturated sands under conventional triaxial stress paths. However, unsaturated soil behavior differs from that of saturated soil because of the presence of a contractile air-water interphase across which a pressure balance is created between the pore-air and pore-water phases within the soil matrix, also known as matric suction [20]. Extensive experimental evidence has clearly demonstrated the paramount influence of matric suction on the stress-strain-strength response of unsaturated soils.

Russell and Khalili [21] were among the first to extend the BS plasticity theory to unsaturated soils within a critical state framework using concept of effective stress and taking into account the particle crushing associated with shearing. The model was calibrated using results from triaxial compression tests on speswhite kaolin reported by Wheeler and Sivakumar [22], as well as triaxial compression and oedometric tests on kurnell sand with no fines reported by Russell [23].

More recently, Morvan et al. [24] presented an extension of Bardet's [7] original BS framework with the aim of simulating post-peak softening of unsaturated soils using 12 constitutive parameters. The extended BS framework allowed them to reproduce, with reasonable accuracy, all the following: 1) increase in shear strength with increasing suction; 2) volumetric collapse or slight rebound upon wetting, depending on mean stress level; and 3) post-peak softening and gradual transition from contractant to dilatant nature of volume change during continuous shearing. However, the calibration and validation of the extended BS framework have been based on a limited set of experimental data obtained from suction-controlled triaxial tests on dense Kurnell sand [23], Jossigny silt [25], and Speswhite Kaolin [26, 27]. These limited studies suggest a need for additional experimental evidence on different types of soils, particularly heavily overconsolidated soils, for a more thorough and conclusive validation of the extended BS framework, a chief motivation for the present work.

2. EXPERIMENTAL PROGRAM

2.1. Test Material

The soil used in the present work is an intermediate geomaterial, hence neither a perfect sand, silt or clay, and tested in a very dense state, thus making it a suitable material to assess the suitability of the extended BS framework introduced by Morvan et al. [24] for overconsolidated, cohesive-frictional soils.

The test material used is classified as a poorly graded silty sand (SM) with clay, according to the Unified Soil Classification System (USCS), with a standard proctor dry density of 1.87 g/cm^3, optimum water content of 12.2% and specific gravity of soil solids, G_s = 2.67. It is well known that the structure and fabric of a test specimen gets altered by the type of compaction method and hence tends to influence the subsequent shear response. In the present work, a stress-controlled approach was used to produce compacted specimen with an artificially induced over-consolidated stress history. Target density of approximately 1.87g/cm^3, corresponding to +2% wet of Proctor optimum, was achieved by compacting each specimen in nine equal lifts, each lift being axially stressed in a split mold to a maximum vertical stress equal to 1600 kPa. The specimen produced had initial voids ratio varying between 0.46-0.48 and approximate air entry value between 8-10 kPa [28].

2.2 Experimental Variables

Strength tests were conducted using a fully-automated double-walled triaxial test equipment that accommodates the essential modifications for unsaturated soil testing, including high-air-entry (HAE) ceramics in the bottom pedestal; pore-water pressure control; pore-air pressure supply via the top cap; and diffused-air flushing assembly, as shown in Fig. 1. A panoramic view of the entire test set-up is shown in Figure 2.

A comprehensive series of saturated and unsaturated consolidated drained (CD) triaxial tests on statically compacted silty sand specimens was performed. The axis-translation technique was used to impose and control matric suction in the range of 0 to 750 kPa (s = 0, 50, 250, 500 and 750 kPa). Three net confining pressures, ($\sigma_3 - u_a$) = 100, 200 and 300 kPa, were applied. Detailed protocols on soil preconditioning, pore-fluid equalization, s-controlled isotropic loading and monotonic shear loading, capability of test equipment to replicate test results and other test related methodology are described by Patil [28].

3. SILTY SAND RESPONSE FROM SUCTION-CONTROLLED TRIAXIAL TESTING

3.1. Effect of Net Confining Pressure at Constant Matric Suction

Test results from suction-controlled CTC (Conventional triaxial compression) tests under constant matric suction, s = 250 kPa, and under three net confining pressures, $(\sigma_3 - u_a)$ = 100, 200 and 300 kPa, are shown in Figure 3. A negative sign in the volumetric strain response represents dilating volume change, while a positive sign indicates compressive type behavior. The convention used to designate the specimen is $CD_{x\text{-}y}$ where "CD" denotes the consolidated drained test; "x" represents the net confining pressure $(\sigma_3 - u_a)$, while "y" represents the imposed constant suction (s).

As expected, Figure 3(a) clearly indicates an increase in stiffness (initial and subsequent tangent modulus), as well as brittleness, with increasing confinement. Post-peak reduction in deviator stress, also known as strain-softening type behavior, was also observed. Moreover, an increase in net confinement pressure is expected to cause higher compression of the specimen and hence suppress the amount of dilation. This is clearly manifested in Figure 3(b), where dilation is significantly suppressed with an increase in confining pressure from 100 to 300 kPa. Such type of stress-strain and volume change response are typical of relatively dense and overconsolidated soils, as is the case of statically compacted silty sands [21, 29]. On the other hand, triaxial tests performed on saturated specimens of compacted silty sand (s = 0) resulted in compressive type behavior. Similar response was observed by previous researchers [21, 30–33].

3.2. Effect of Matric Suction at Constant Net Confining Pressure

Results from suction-controlled CTC tests conducted on compacted silty sand specimens, under varying matric suction states, s = 0, 50, 250, 500, and 750 kPa, and constant net confining pressure, $(\sigma_3 - u_a)$ = 100, are shown in Figure 4. In these figures, the change from strain-hardening type behavior, as seen in saturated specimens (s = 0), to strain-softening type response, due to the introduction of matric suction in all unsaturated specimens, is readily manifest. The amplitude of strain-softening increases and the specimen fails at lower axial strain, while taking less axial strain to reach critical state, with increasing matric suction, as shown in Fig. 4(a).

Figure 4(b) shows a rather drastic change in volumetric response, from compressive to dilational type behavior, with the imposition of a matric suction of magnitude as low as 50 kPa. Matric suction tends to increase the initial stiffness and the apparent cohesion between individual soil particles so that the particles all together during shearing tend to ride over one another instead of undergoing slippage, resulting in an increase in volume (i.e. dilation). However, the rate of dilation decreases as the air-water menisci gets destroyed with continued shearing beyond peak stress, and the specimens get softened back to critical state, at which point they are expected to exhibit only shear deformations (plastic flow) without further change in strength or volume. All the test specimens were observed to develop multiple shear bands while simultaneously bulging at the center as the shearing continued [28].

In addition, experimental results indicate that an increase in matric suction has more influence on peak shear stress than on critical shear stress. Continued shearing beyond peak shear stress tends to weaken and subsequently destroy the matric suction effect around soil grains due to perturbation caused to air-water menisci interface. This phenomenon can be attributed to the observed experimental softening of stress, the amplitude of which depends largely on the magnitude of suction imposed, ultimately resulting in peak strength being more affected by matric suction than by critical or large strain strength.

4. CONSTUTIVE MODELING OF OVERCONSOLIDATED SILTY SAND RESPONSE

4.1. Bounding Surface Model: General Framework

The original framework of the Bounding Surface Model (BSM), particularly the one introduced by Morvan et al. [24] for unsaturated soils, has proved reasonably efficient in modeling both compressive volumetric response and strain-induced hardening. These behaviors are typically observed in normally and lightly overconsolidated soils. However, this BSM framework also has the potential to reproduce dilational volumetric response and strain-induced softening that are typically observed in heavily overconsolidated soils subjected to suction-controlled monotonic shearing. Furthermore, it is possible with BSM to capture the gradual transition between elastic and elasto-plastic soil response. The chief motivation for present work was to finetune and refine the original BSM to predict the stress-strain and volume change response and compare it with the one obtained via suction-controlled triaxial testing from present research. Since the tests to be modeled are performed at constant suction and the volumetric variations of the sample are low, we chose not to use the fully coupled version taking into account water retention curve hysteresis and its dependency on void ratio.

4.2. Effective Stress: Definition and Radial Mapping

The effective stress, σ', is expressed in terms of the equivalent pore pressure, π, as defined by Pereira et al. [34], and hence is given as follows:

$$\sigma' = \sigma + \pi(s)\mathbf{1} \qquad \text{with } \pi(s) = -u_a + sS_r + \frac{2}{3}\int_{S_r}^{1} s(S)dS \tag{1}$$

As previously mentioned, the model introduced by Morvan *et al.* [24] to simulate unsaturated soil response is essentially an extension of the BS model postulated by Bardet [7] for saturated soils, and hence is based on the bounding surface theory within a critical state framework. Therefore, due consideration is given to the existence of a limit state line (LSL) in the p', q plane that defines an upper bound to the stress ratio $\eta_p = q/p'$, as illustrated in Fig. 5. The plastic modulus is forced to be dependent on the distance δ between the current stress state σ' and its image-point, $\bar{\sigma}$ obtained by projection on the distance δ between the current stress state σ' and its image-point, obtained by projection on a surface called bounding surface, as shown in Figure 5.

Radial mapping technique [7], also illustrated by Yu [29], is used to define the image-point $\bar{\sigma}'$ of the actual stress σ' on the bounding surface BS, via a translation vector, by extending the line linking the origin to the point σ′ until it cuts the bounding surface (Fig. 5). Further details on the classic BS formulation have been presented by Dafalias and Herrmann [5], Crouch et al. [35], Manzari and Dafalias [36], Russell and Khalili [21], and Yu [29].

4.3. BSM Framework in Triaxial Stress Space

Classic triaxial test variables are used for cylindrical symmetry and triaxial stress space to define the mean net stress (p'), deviatoric stress (q), peak state line (PSL) slope (η_p), and critical state line (CSL) slope (M), which are expressed as follows:

$$p' = \frac{1}{3}(\sigma'_1 + 2\,\sigma'_3), \;\; q = \sigma'_1 - \sigma'_3, \qquad \eta_p = \frac{q_p}{p'_p}\,, \qquad M = \frac{q_{cs}}{p'_{cs}} \tag{2}$$

Different variables/parameters used are defined in the list of symbols at the end of paper. In this work, peak state line (PSL) is used exchangeable to limit sate line (LSL), as shown in Fig. 5. A third line, identified as the characteristic state line (CL), which marks a transition from contractant to dilatant volume change behavior during initial stages of shear loading, is also postulated.

The conjugated variables in p', q plane is volumetric and deviatoric strains, ε_p and ε_q, which are respectively defined as follows:

$$\varepsilon_p = \varepsilon_1 + 2\,\varepsilon_3, \qquad \varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \tag{3}$$

The total strain is assumed to be divided into elastic and plastic parts:

$$\varepsilon_p = \varepsilon_p^e + \varepsilon_p^p, \qquad \varepsilon_q = \varepsilon_q^e + \varepsilon_q^p \tag{4}$$

The BS yield function represents an ellipse-shape yield surface in p':q plane (Fig. 5), beyond which plastic compression occurs on account of increased stress or decreased suction, and is defined as follows:

$$f\left(\bar{p}', \bar{q}, \varepsilon_p^p, s\right) = \left(\frac{\bar{p}' - A_\pi}{\rho - 1}\right)^2 - \left(\frac{\bar{q}}{M}\right)^2 - {A_\pi}^2 \tag{5}$$

The stress image obtained by the radial mapping, as shown in Fig. 5, can be defined as follows:

$$\bar{p}' = \gamma A_\pi, \quad \bar{q} = \gamma x M A_\pi, \qquad x = \frac{q}{Mp' + q_0} \tag{6}$$

$$\gamma = \frac{(1 + (\rho - 1)\sqrt{1 + x^2\rho(\rho - 2)}}{1 + (\rho - 1)^2 x^2} \tag{7}$$

Employing the regular concept and the definitions of plastic multiplier ζ and plastic modulus H_b, the following can be established:

$$\frac{\partial f}{\partial \varepsilon_p^p} \frac{\mathrm{d}\varepsilon_p^p}{\left\|\frac{\partial f}{\partial \bar{\sigma}}\right\|} = -H_b d\zeta \tag{8}$$

The gradient of f contains 3 terms, i.e. the partial derivatives with respect to $\bar{p}'$, $\bar{q}$, and s:

$$\frac{\partial f}{\partial \bar{\sigma}'} = \left\{\frac{\partial f}{\partial \bar{p}'}; \frac{\partial f}{\partial q}; \frac{\partial f}{\partial s}\right\}^{\mathrm{T}} \tag{9}$$

After few simplifications, the 3 components of the exterior normal at the image-point can be expressed as follows:

$$\eta_p = \frac{\partial f}{\partial \bar{p}'} \Big/ \left\| \frac{\partial f}{\partial \bar{\sigma}} \right\| = \frac{M(\gamma - 1)}{g} \tag{10}$$

$$\eta_q = \frac{\partial f}{\partial \bar{q}} \Big/ \left\| \frac{\partial f}{\partial \bar{\sigma}'} \right\| = \frac{(\gamma x(\rho - 1))^2}{g} \tag{11}$$

$$\eta_s = \frac{\partial f}{\partial s} \Big/ \left\| \frac{\partial f}{\partial \bar{\sigma}} \right\| \tag{12}$$

with: $g = \| \frac{\partial f}{(\partial \bar{\sigma}')} \| \frac{M(\rho - 1)^2}{2A_\pi}$ (13)

When the stress point is on the bounding surface, the plastic modulus is given by:

$$H_b = \frac{1 + e_0}{\lambda(s) - \mathrm{\kappa}} \frac{Al_1(s)}{g^2} M^2(\gamma - 1)[\gamma + \rho(\rho - 2)] \tag{14}$$

where κ is the volumetric deformability and e_0 is the initial void ratio. On the other hand, if the stress point lie within the bounding surface, an additional term H_f is added to H_b. The plastic modulus H is then expressed as follows:

$$H = H_b + H_f \tag{15}$$

H_f is related to the plastic modulus H by the distance δ between the current stress state σ' and its projected image-point $\bar{\sigma}'$ on the bounding surface, as illustrated in Fig. 5, and is expressed as follows:

$$H_f = \frac{1 + e_0}{\lambda(s) - \mathrm{\kappa}} \frac{\delta}{\delta_{max} - \delta} h_0 p' \frac{\eta_p - \eta}{M} \tag{16}$$

where h_0 is a material parameter.

Inclusion of hardening of the unsaturated soil due to an increase in matric suction and plastic volumetric strains is essential for the development of a consistent hardening rule. The plastic flow is assumed to take the same direction as the onset of yielding by adopting an associative flow rule relating the incremental plastic components of volumetric and shear strains, which are expressed as follows:

$$d\varepsilon_p^p = \left[\frac{1}{H}(n_p dp' + n_q dq + n_s ds)\right] n_q \tag{17}$$

$$d\varepsilon_q^p = \left[\frac{1}{H}(n_p dp' + n_q dq + n_s ds)\right] n_q \tag{18}$$

while the elastic strains are defined by the classic relations:

$$d\varepsilon_p^e = \frac{\kappa}{(1+e_0)}\frac{dp'}{p'}\ , \qquad d\varepsilon_q^e = \frac{2(1+\nu)}{9(1-2\nu)}\frac{\kappa}{(1+e_0)}\frac{dq}{p'} \tag{19}$$

where v is the poisson's ratio.

Considering the present experimental data, the suction-dependence of A_π can be expressed as follows:

$$A_\pi = A_\pi\left(\varepsilon_p^p, s\right) = l_1(s)A\left(\varepsilon_p^p\right) + l_2(s) \tag{20}$$

$$l_1(s) = 1 + k_1(S_r s - s_e)\ for\ s > s_e, \qquad l_2(s) = 0 \tag{21}$$

where, S_r is the degree of saturation. In order to ensure continuity at full saturation state and be able to reproduce wetting induced collapse curve, the following conditions must be satisfied:

$$\forall s\ \leq\ s_e: l_1(s) = 1, and\ l_2(s) = 0 \tag{22}$$

$$\forall s\ \geq s_e: l_1'(s) \geq 0, and\ l_2'(s) \geq 0, and \qquad \rho(A\left(\varepsilon_p^p\right)l_1'(s) + l_2'(s)) > \frac{d\pi(s)}{ds} \tag{23}$$

The slope of CSL in the e-log p' plane is assumed to be identical to that of the normal compression line (NCL) at constant suction, and is defined as follows:

$$\lambda(s) = \lambda_0 - k_3(S_r s - s_e) \quad for\ s > s_e \tag{24}$$

4.4. Essential BSM Constitutive Parameters

The modified BS model postulated in the present work requires 12 material parameters, 8 of which are necessary to define the behavior at full saturation: (1) Two elastic constants, ν and κ; (2) Six constants for plastic behavior, out of which three are required to determine the position and the shape of the bounding surface ρ, M and A_0, two for the plastic modulus, η_p and h_0, and one for the volumetric compressibility, λ_0. Four other constants are needed to account for suction: (1) Two constants, s_e and α, to define the water retention curve, information required to determine the equivalent pore pressure so that the effective stress can be obtained; (2) One constant, k_1, to account for suction effects on the hardening parameter A_π, and (3) the last one, k_3, to define the function $\lambda(s)$.

Systematic calibration procedure was followed to extract BS model parameters for compacted silty sand from conducted experiments. First, the parameters s_e and α are needed to be able to obtain effective stress. To achieve this, water retention curve in term of degree of saturation is used. These parameters are those given by Brooks and Corey [37]. M and η_p that define critical state and limit state are determined by plotting results of saturated triaxial tests in p',q plane. Volumetric stiffness $\lambda(s)$ was obtained by plotting isotropic tests results in e-log p' plane, which is necessary to obtain the initial value λ_0, and its variation with suction (k_3). The variation of A_π with suction is determined using loading collapse curves.

Only a few types of compacted geomaterials have been reported as being modeled with the BS framework as summarized in the above sections. Table 1 summarizes the values of all essential BS model parameters calibrated by previous researchers, along with those from present research. In the present work, results were thoroughly analyzed to gain critical insight into some of the most essential elastoplastic features of compacted intermediate geomaterials under controlled suction states, including the effect of suction on yield stress, apparent cohesion, tensile strength, critical state line, post-peak softening and strain-induced dilatancy under suction-controlled monotonic shearing.

It is also worth noting, however, that the mechanical properties of unsaturated soils can be greatly influenced by repeated wetting and drying cycles, commonly termed as hydraulic hysteresis. This hydraulic hysteresis could be taken into account by including few modifications to the BS model, as proposed by Morvan et al. [38]. Nevertheless, in case of sandy soils, such as the test soil used in the present work, hydraulic hysteresis effects could be reasonably neglected.

4.5. Loading Collapse Locus using BS Model

Unsaturated soil could undergo volumetric collapse or slight rebound upon wetting, depending on the mean stress level. The functions $l_1(s)$ and $l_2(s)$, along with $A_\pi\left(\varepsilon_p^p, s\right)$, define the classic loading collapse (LC) curve in the p', s plane, as shown in Figure 6, including the limiting values of p' (p'_{lim}) on the bounding surface. The BSM predicted LC curve ensures continuity at saturation and satisfies conditions in Eq. (24).

In the BSM framework, and in contrast with the original BBM presented by Alonso et al. [2], the stress state, while moving forward along an imposed triaxial stress path, can cross the CSL and hence access the domain $\eta > M$. However, the plastic modulus "H" changes sign, somewhere between the CSL and the LSL, to satisfy the condition $M < \eta < \eta_p$. This means that the stress state can never reach LSL. The parameter "η" is the lower limit of η_{peak} and, if properly chosen, imparts the ability to reproduce the post-peak decrease in stress (softening). The value of η can then be obtained by plotting the experimental values of peak deviator stress and corresponding mean net pressure at s = 0, 50, 250, 500, and 750 kPa under three net confining pressures, $(\sigma_3 - u_a)$ = 100, 200 and 300 kPa, as shown in Figure 7. Thus, the best-fit line in Figure 7 defines the lower value of limit state line slope. A value of η = 1.77 was obtained and it was rounded off to η_p = 1.8. This value represents an upper bound to the physically possible domain for stress.

4.6. Parametric Investigation of BS Model Predictions

Figure 8 shows the results of a parametric investigation of BS model predictions with five different values of slope η_p for triaxial testing at constant suction, s = 50 kPa, and net confining pressure, $(\sigma_3 - u_a)$ = 300 kPa. It can be clearly observed that the BS model is capable to simulate a rather smooth transition of the nature of soil volume change, from contractant to dilatant, as well as the corresponding transition of soil response from initial hardening to post-peak strain softening. The parametric study also identifies a limiting case with $\eta_p = M = 1.3$, beyond which any further increase in η_p will initiate dilatancy and the stress-strain curve will exhibit a distinct peak stress.

4.7. Barcelona Basic Model: Essential Features

Alonso et al. [2] postulated a unified, critical state based, constitutive framework for unsaturated soils by extending the modified Cam-Clay model [39], from saturated to unsaturated form, using suction as an independent stress variable, while introducing the concept of the loading-collapse (LC) yield surface. Consequently, constitutive parameters postulated by both the BBM and BSM frameworks were experimentally calibrated from the series of suction-controlled triaxial tests

(Tables 1 and 2), and then used for predicting compacted silty sand suction-controlled axisymmetric shearing response at constant matric suction states varying from 50 to 750 kPa.

Further details regarding the explicit, step by step integration of BBM constitutive relations are given by Macari et al. [40] and Hoyos et al. [41]. Also, implementation of BBM theory to calibrate the essential constitutive model parameter via experimental tests can be obtained from Patil [28]. Complementary simulations using the Barcelona Basic Model (BBM) have also been included, alongside BS model predictions, in order to get further enlightening insights into some of the main limitations and challenges facing both frameworks within the context of the experimental evidence resulting from the present research effort.

5. IMPLICIT INTEGRATION OF BSM CONSTITUTIVE RELATIONS

The present section illustrates the adopted protocol for implicitly integrating all the BSM constitutive relations as summarized by equations (1)-(24) presented in the earlier sections. Calibrated BSM parameters from Table 1 are used in order to simulate silty sand behavior for matric suction, s = 250 kPa, and net mean pressure, $(\sigma_3 - u_a)$ = 300 kPa. Figures 9(a) and 9(b) show the corresponding stress-strain and volume change simulations along with observed experimental response. Figure 9(c) clearly depicts the suction-induced growth experienced by the initial yield surface of an unsaturated soil, as compared to the saturated case. Further, it illustrates the progression of yield surfaces [i.e. (1)-(6)] as the specimen is loaded monotonically under consolidated drained condition along an 1H:3V stress path, i.e. conventional triaxial compression (CTC) stress path, in p-q space. The evolution of yield surface is largely governed by change in plastic volumetric strains.

Point "1" corresponds to the initial state of a fictitious BS surface prior to shear loading, with its equation calculated from the post-consolidation void ratio and mean net stress using Bardet [7] equation, as expressed below:

$$A_{\pi} = [1 + k_1(sSr - s_e)]A_0 e^{\left(\frac{\Gamma - e - \kappa ln\,(p)}{\lambda - \kappa}\right)} \tag{25}$$

where e = current voids ratio, p = mean net pressure, and A_0 = unit pressure. The evolution of the variable A_{π} in Eq. (5) is described by the classic Cam-Clay hardening rule [38]. Bounding surface equation (Eq. 5) depends on "A" as well as "A_{π}", so it permits to obtain mechanical hardening as well as suction hardening,

$$\mathrm{d}A = \frac{1+e_0}{\lambda_0 - \kappa} A \mathrm{d}\varepsilon_p^p \quad and \quad A_{\pi} = [1 + k_1(sSr - s_e)]A(\varepsilon_p^p) \tag{26}$$

For the sake of better understanding, the typical stress-strain curve is embedded to the right of the BS yield curves in Fig. 9(c). As the specimen is loaded, the induced plastic strains cause the bounding surface to move during plastic flow, but the BS must always envelope the current state. The bounding surface expands in size isotropically as specimen is loaded between stress state 1-2-3, inducing compressive volumetric strains. Point 3 on the stress-strain curve identifies the onset of dilation and is also known as the characteristic state (i.e. CL). It marks a transition state of volumetric behavior from contracting to dilating. The yield surface (BS) attains its maximum size at characteristic state (point 3). As the loading continues further beyond characteristic point 3, the stress state continues to move upwards towards peak failure line. However, it cannot cross LSL (point 4), as explained earlier. Concomitantly, the test soil starts dilating, and the BS starts to shrink in size as the plastic volumetric strains starts to decrease.

It should be noted that the observed peak dilatancy is attained far before reaching the peak shear strength. Upon reaching the peak stress state, the specimen undergoes strain-softening with further increase in deviator stress and the yield surface continues to shrink in size between points 4 and 5, at a rate faster than corresponding decrease in stress state. According to Bardet [7] theory for saturated soil, the stress state continues to decrease while the yield surface continues to shrink further, until they both reach a common point, and thereafter they both move together towards and stop at a point (point 6) that represents critical state, which correspond to a state of no volume change, no stress change, and hence zero plastic volumetric strains. In these simulations, critical state gives an asymptote to the stress path but is not reached. The same goes for the bounding surface (BS), which keeps shrinking slowly but does not reach the stress state along stress path, even with imposed axial strains of up to 40%.

6. BSM AND BBM PREDICTIONS OF COMPACTED SILTY SAND RESPONSE

BSM and BBM simulations were compared with the experimental results obtained from the series of suction-controlled CD-triaxial shear tests. All specimens were sheared along CTC stress paths. Figs. 10-14 show comparisons between BSM and BBM predictions and experimentally observed deviator stress vs. axial strain response of compacted silty sand from fully drained (constant suction) CTC tests conducted at four different values of matric suction, s = 0, 50, 250, 500 and 750 kPa, and for initial values of net mean stress, $p = (\sigma_3 - u_a)$ = 100, 200, and 300 kPa, respectively.

In general, no close agreement is observed between experimental behavior and BBM predictions for overconsolidated silty sand, primarily given the largely brittle and dilatant nature of the test soil before it reaches critical state. For hardening materials, continued shearing along a suction-controlled CTC stress path (Fig. 5) is expected to cause the elliptical yield surface in p:q plane to move outward (or increase in size) from the current point, while it would move inward (or shrink in size) for softening materials. The relative position of the current stress state with respect to the CSL governs whether the material sustains plastic dilatancy or contractancy.

The original BBM framework, however, does not contemplate a stress state that lies beyond the CSL at which all induced deformation is plastic, since the peak stress is always assumed to be reached at critical state. Therefore, it is not expected to be suitable for reproducing the transition from initial contractancy to dilatancy, and hence the post-peak strain softening commonly observed in dense or overconsolidated geomaterials (Figs. 11-14). For this reason, only BBM predictions of stress-strain response are shown in Figures 10-14, obviating the corresponding volume change response.

In effect, although predictions of deviatoric stress (end values) at critical state still proved to be reasonably close to those experimentally observed (Figs. 10-14), BBM predictions considerably deviate from the stress-strain response of compacted silty sand, particularly at higher matric suctions and net confining pressures, which are precisely the stress state for which the test soil exhibits largest post-peak softening, accompanied by significant dilation. From a qualitative standpoint, however, reasonably good predictions are generally observed for most of the initial shearing stage, up to about 1-2% shear strain, and at considerably large values of shear strain, i.e., critical state.

On the contrary, the BSM is able to reproduce the stress-strain and volume change response of compacted silty sand, under both saturated and unsaturated conditions, with reasonable accuracy. It is worth noting that saturated silty sand samples showed compressive type volume change response, which turned into dilating type as soon as the soil was subjected to a matric suction as low as 50 kPa. The BSM was able to capture this drastic transition in volume change quite smoothly. Furthermore, the BSM is able to simulate the increase in dilation with increasing matric suction, rather smoothly as well, and with reasonable accuracy, as shown in Figs. 10(c)-14(c). This transition is possible because of the term added to the plastic modulus when the stress point lies inside the surface (Eqs.15 and 16).

On the other hand, saturated silty sand samples showed hardening type stress-strain response, which quickly changed to post-peak softening type behavior under suction-controlled conditions (s = 50, 250, 500 and 750 kPa). Likewise, the BSM was able to capture this transition rather smoothly and successfully. The increase in magnitude of post-peak softening with increasing matric suction is also closely captured by the BSM, as shown in Figs. 11(a)-14(a). This close prediction of post-peak strain softening can be attributed to the incorporation of two state lines into the BSM framework: the commonly used CSL (critical state line) and the LSL (limit state line), as shown in Fig. 5. The stress path can cross the CSL, but the LSL limits the accessible domain and whether it is crossed or not, the CSL will be the asymptote of the stress path.

Figs. 11(a & b)-14(a & b) indicate that although the BBM is not able to closely capture the typical stress-strain response, it performs relatively well in predicting deviator stress at peak and critical state under low net confining pressure irrespective of magnitude of matric suction applied. On the other hand, BSM performs relatively well in predicting peak and critical deviator stress under high net confining pressures irrespective of magnitude of matric suction applied.

Finally, good correlations were obtained, for the tested suction and confining pressure range, between experimental and the extended BSM predicted deviator stress values, both at critical state failure (R^2 = 0.91), and peak state failure (R^2 = 0.85), as shown in Figure 15(a) and (c). Although, relatively better correlations were obtained with BBM as compared to BSM at critical state failure (R^2 = 0.96), and peak state failure (R^2 = 0.93) as shown in Fig. 15(b) and (d), it cannot closely reproduce the continuous stress-strain and volume change response (Figs. 10-14). The slope of the critical state line in p'-q plane did not significantly changed with suction, and thus can be considered to remain virtually constant and non-sensitive to changes in suction. Figure 16 shows the variation of the slopes of critical state lines in p'-q plane for different soils as reported from current and previous research works [22, 42–44].

During suction-controlled experiments, the practical range of applying matric suction inside soil specimen is only up to 1500 kPa, due to limitations of ceramic disk in using axis-translation technique [20]. In addition, increase in magnitude of induced suction, well beyond residual suction, will dramatically escalate the magnitude of drop in post-peak strength and hence softening upon monotonic shearing as well as reduce the overall amount of dilational volume change [28].

Incorporating such a dramatic variation in stress-strain response into modeling, that alters from strain-hardening for saturated soil to strain-softening with introduction of matric suction and which further tends to show increase in amplitude of softening with further increase in total suction for test soil (i.e. silty sand), poses a great challenge. Currently, the authors are investigating into experimental response at high suction to improvise the existing model, especially to incorporate additional parameters to reflect high suction impact on response, to enable possible better predictions, particularly above residual suction, where any change in moisture content is merely due to vapor-phase exchange.

7. SUMMARY AND CONCLUSIONS

12 consolidated drained (CD), conventional triaxial compression (CTC) tests were conducted on statically compacted specimens of unsaturated silty sand (SM) under four suction-controlled

conditions and three net confining pressures. Experiments were performed in a fully-automated double-walled triaxial test system and target matric suction states varying from 50 to 750 kPa was induced via the axis-translation technique. Experimental results from present research showed persuasive evidence of augmentation in shear strength, stiffness, yield strength, tensile strength, post-peak strain-softening and dilatancy with increase in matric suction. A distinct, experimental compaction induced loading-collapse (LC) locus was readily identified in the p:s plane and comparison was made with BS model predicted LC locus.

Essential constitutive parameters postulated by the BBM and the extended BS model framework were then calibrated and used for prediction of peak deviator stress at matric suction states varying from 50 to 750 kPa. There were no good agreements between the observed experimental and predicted BBM stress-strain responses, especially the post-peak softening behavior. BBM predictions, however, hold reasonably well mostly during the early shearing stage (i.e. 1-2% axial strain) and at higher values of shear strain, i.e. critical state.

On the other hand, the extended BS model was able to simulate the stress-strain and volume change response reasonably well. The BS model was able to capture smoothly the transition of stress-strain response from strain-hardening to post-peak strain softening along with the contemporaneous transition of volume change from compressive to dilatant type on account of suction with reasonably well captured simulations. The major advantage of BS model is its flexibility to allow for shrinking of yield surface based on value of ($\eta_{\mathrm{peak}} - \eta_{\mathrm{critical}}$), to accommodate the post-peak softening behavior, which otherwise, was not possible with classical model such as BBM. In general, irrespective of the value of matric suction applied, the BBM performs relatively well in predicting response at peak and critical state failure under low net confining pressure while the extended BSM performs relatively well under high net confining pressures.

With additional testing in future, the current version of generalized BS model framework that seems to have promising applications to unsaturated soil mechanics and shear strength applications could be further generalized and made more flexible to accommodate experimental response from more soil types applied over broader suction range.

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List of Tables

Table 1. Essential BSM parameters calibrated from past and present works

Material constants	κ	ν	ρ	M	Γ	η_p	h_0	s_e	α	k_1	k_3	λ_0
Units	-	-	-	-	-	-	-	kPa	-	-	-	-
	0.01	0.125	2.2	1.2	0.73	1.2	2	15	2.1	2	0.05	0.1425
**	0.006	0.3	1.7	1.6	0.95	1.9	5	6	0.2	5	0.05	0.155
***	0.05	0.2	2.2	0.72	1.1	0.74	5	-	0.066	0.08	0.1907	0.19
****	0.008	0.3	1.7	1.3	0.7	1.8	5	8	0.3	2.23	0.003	0.04

Notes: * – Cui and Delage [25]; Triaxial compression test on compacted jossigny silt and BS Model parameters extracted by Morvan et al. [24].

** – Russell [23]; Drained triaxial test on Kurnell sand and BS Model parameters extracted by Morvan et al. (2010).

*** – Raveendiraraj [27] and Gallipoli et al. [26]; Triaxial test on speswhite kaolin and BS Model parameters extracted by Morvan et al. (2010).

**** – CD triaxial test on poorly graded silty sand (SM) with clay; Present research.

Table 2. Essential BBM Parameters for current test material

Parameter	Current work	Units
$\lambda(0)$	0.020	-
κ	0.0015	-
β	2.000	MPa^{-1}
r	0.260	-
p^c	0.046	MPa
G	25.00	MPa
M	1.420	-
k	0.223	-
$p_o(0)$	0.070	MPa

List of Figures

Figure 13. Experimental and BSM/BBM predictions of unsaturated silty sand response at matric suction, s = 500 kPa, and different net confining pressures(a), (b) stress-strain response, and (c) volume change response

Figure 14. Experimental and BSM/BBM predictions of unsaturated silty sand response at matric suction, s = 750 kPa, and different net confining pressures: (a), (b) stress-strain response, and (c) volume change response

Figure 15. Comparison of predicted and measured values of deviator stress at peak and critical state failure as postulated by the BSM and BBM framework

Figure 16. Current and previously reported values of critical state line slope variation with matric suction

Following is the list of symbols used in this paper:

π	equivalent pore pressure
η	stress ratio q/p
η_p, η_{cr}	maximum stress ratio at peak state and critical state failure
$\bar{\sigma}$	projection of stress state i.e., image point on bounding surface
δ	distance between stress state and image stress
p, q, p'	mean pressure, deviator stress, and net mean pressure
$\bar{p}, \bar{q}$	mean pressure and deviator stress at image point
p'_{cr}, q_{cr}	mean net pressure and deviator stress at critical state
ε_i $i = 1, 2, 3$	components of strain in triaxial space
$\varepsilon_p, \varepsilon_q$	volumetric and deviator strain
$\varepsilon_p^e, \varepsilon_q^e$	elastic volumetric and deviator strain
$\varepsilon_p^p, \varepsilon_q^p$	plastic volumetric and deviator strain
A, A_π	position of bounding surface summit on the p-axis
K_p	elastic bulk modulus
e, e_c, e_0	present, critical, and initial voids ratio
G, K_q	elastic shear moduli
g	amplitude of gradient of bounding surface at image stress
H, H_b, H_δ	plastic moduli
K_1, k_2, K_3	material constants to account for unsaturated state
l_1, l_2	material constants to account for unsaturated state
M, M_π	position of bounding surface summit on the p-axis
n_p, n_q, n_s	components of unit vector normal to bounding surface at image point
s	suction
S_1	degree of saturation
x, y	normalized stress ratio, variable defining image position
ζ	plastic multiplier
α, s_e	constants defining water retention curve
κ	elastic volumetric deformability
$\lambda_0, \lambda, \Gamma, q_0$	values determining position of the critical state
v	elastic poisson ratio
ρ	aspect ratio of ellipse
σ_i $i = 1, 2, 3$	components of stress in triaxial space

σ_i' $i = 1, 2, 3$ components of effective stress in triaxial space