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HAL Id: hal-02022622
https://uca.hal.science/hal-02022622

Submitted on 18 Feb 2019

Spreading Factor Allocation Strategy for LoRa Networks under Imperfect Orthogonality

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Abstract—Low-Power Wide-Area Network (LPWAN) based on LoRa physical layer is envisioned as one of the most promising technologies to support future Internet of Things (IoT) systems. LoRa provides flexible adaptations of coverage and data rates by allocating different Spreading Factors (SFs) to end-devices. Although most works so far had considered perfect orthogonality among SFs, the harmful effects of inter-SF interferences have been demonstrated recently. Therefore in this work, we consider the problem of SF allocation optimization under co-SF and inter-SF interferences, for uplink transmissions from end-devices to the gateway. To provide fairness, we formulate the problem as maximizing the minimum achievable average rate in LoRa, and propose a SF allocation algorithm based on matching theory. Numerical results show that our proposed algorithm enables to jointly enhance the minimal user rates, network throughput and fairness, compared to baseline SF allocation methods.

Index Terms—LoRa, Spreading Factor, Imperfect Orthogonality, Resource Allocation Optimization, Matching Theory

Fig. 1. LoRa system, with end-devices transmitting simultaneously

I. INTRODUCTION

Tens of billions of Internet of Thing (IoT) devices are expected by 2020, for enabling groundbreaking applications such as smart cities, intelligent transportation systems or harsh environment monitoring. Many of these IoT Wireless Sensor Networks (WSNs) will strongly rely on Low-Power Wide-Area Network (LPWAN) technologies such as Long Range (LoRa) [1, 2]. LoRa is one of the most prominent physical layer technologies for LPWAN, as it operates in the ISM unlicensed bands and enables flexible adaptations of transmission rates and coverages under low energy consumption [3]. LoRa is based on Chirp Spread Spectrum (CSS) modulation, where each chirp encodes 2^m symbol values, for Spreading Factor (SF) m. On top of LoRa physical layer, LoRaWAN defines the MAC layer protocol standardized by LoRa Alliance [4]. The LoRaWAN architecture is a star-like topology, where end-devices communicate with gateways over several channels based on ALOHA mechanism, with duty cycle limitations [1]. In LoRaWAN, six SFs ranging from $m = 7$ to 12 are used, where smaller SFs provide higher data rates but reduced ranges, while larger SFs allow longer ranges but lower rates [2].

One of the main issues in LoRa and LoRaWAN is the throughput limitation: the physical bitrate varies between 300 and 50000 bps [4]. In addition, collisions are very harmful to the system perform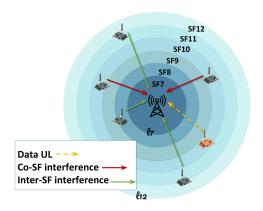ance as the LoRa gateway is unable to correctly decode simultaneous signals sent by devices using the same SF and the same channel. Such interferences will be referred to as co-SF interferences. Although SFs were widely believed to be orthogonal among themselves, some recent studies have shown that this is not the case by experimentally evaluating the effects of inter-SF interferences [5, 6]. Thus, authors in [7] have analyzed the achievable uplink LoRa throughput under imperfect SF orthogonality, and have demonstrated the harmful impact of both co-SF and inter-SF interferences on the overall throughput.

In order to improve the LoRa system performance, a number of works have proposed resource optimization methods [8–10]. However, most papers so far have assumed a perfect orthogonality among SFs. In particular, the authors in [8] design a channel and power allocation algorithm, that maximizes the minimal rate. However, the allocated channels are perfectly orthogonal, and no SF allocation nor SF-dependent rates were not considered, despite the strong dependency of the rate to SFs. In addition, the solution of [8] requires instantaneous Channel State Information (CSI) feedback, which is not adapted to LoRa networks due to their energy consumption limitations [4]. In [9], a heuristic SF-allocation is proposed, where users with similar path losses are simply assigned to the same channel, then to each SF according to their distance to the gateway. Although the issue of inter-SF interferences was highlighted, it was ignored in their proposed solution. In [10], a method for decoding superposed LoRa signals using

the same SF was proposed.

Therefore, in this work, we consider the issue of SF allocation optimization under both co-SF and inter-SF interferences. In particular, we focus on the problem of maximizing the minimum achievable short-term average rate in the uplink, whereby short-term average rate is defined as the average rate over random channel fadings, but given a fixed position of end-devices. This metric is especially suited for LoRa networks, since the end-devices will likely be fixed for a certain period of time (at least for a few seconds) in many applications, and their positions known at the gateway, as in the conventional signal-strength-based SF allocation method [4]. Our focus here is the max-min fairness in terms of throughput, without taking into consideration the energy of end-devices. As LoRaWAN enforces a limited duty-cycle, the energy consumption of each end-devices remains limited. Firstly, we formulate our optimization problem by modeling the achievable uplink short-term average rate under co-SF and inter-SF interferences. Next, given the mathematical intractability of the optimization problem, we propose an SF-allocation algorithm based on matching theory. We show its stability and convergence properties, and analyze its computational complexity. Finally, numerical results demonstrate that, compared to baseline SF-allocation schemes, our proposed method not only provides larger minimum rates, but also jointly improves the network throughput and fairness level.

II. System Model

We consider an uplink LoRa system with a single gateway located at the center of a cell of radius R km and N end-devices uniformly distributed and simultaneously active as depicted in Figure 1. End-devices transmit on the same channel c of bandwidth BW, with a power $P_{\max}$ and a duty cycle of 100%. We denote the set of SFs by $\mathcal{M} = \{7, 8, \ldots, 12\}$, where $M = |\mathcal{M}| = 6$. Each SF_m, $m \in \mathcal{M}$, has a data bit-rate equal to R_m [3],

$$R_m = \frac{m \times CR}{\frac{2^m}{BW}}, \qquad (1)$$

where the Coding Rate is given in [3] as $CR = \frac{4}{4+n}$ with $n \in \{1, 2, 3, 4\}$.

The uplink instantaneous Signal-to-Noise Ratio (SNR), γ_{nm}, for end-device n at SF_m is given by [7],

$$\gamma_{nm} = \frac{P_{\max} |h_n|^2 \mathrm{A}(f_c)}{r_n^\alpha \sigma_c^2}, \qquad (2)$$

where h_n is the channel gain between n and the gateway, $\mathrm{A}(f_c) = (f_c^2 \times 10^{-2.8})^{-1}$ is the deterministic loss, f_c is the carrier frequency, r_n the distance from end-device n to the gateway, α the path loss exponent and $\sigma_c^2 = -174 + \mathrm{NF} + 10\log(BW)$ [dBm] is the Additive White Gaussian Noise (AWGN) where NF is the receiver Noise Figure. Assuming Rayleigh fading channels, the SNR γ_{nm} is modeled as an exponential random variable with mean $\overline{\gamma}_{nm} = \frac{\mathrm{A}(f_c) P_{\max}}{r_n^\alpha \sigma_c^2}$.

SF m	Bit-rate [kb/s]	Receiver sensitivity [3] [dBm]	Reception thresh. $\theta_{\mathrm{rx}\,m}$ [dB]	InterSF thresh. [11] $\tilde{\theta}_m$ [dB]	Distance ranges
7	5.47	-123	-6	-7.5	$0\text{-}l_7$
8	3.13	-126	-9	-9	$l_7\text{-}l_8$
9	1.76	-129	-12	-13.5	$l_8\text{-}l_9$
10	0.98	-132	-15	-15	$l_9\text{-}l_{10}$
11	0.54	-134.5	-17.5	-18	$l_{10}\text{-}l_{11}$
12	0.29	-137	-20	-22.5	$l_{11}\text{-}l_{12}$

TABLE I
LoRa Characteristics at BW=125kHz [7]

The area covered by each SF is given by the distance ranges in Table I with [7],

$$l_m = \mathrm{e}^{\frac{1}{\alpha} \times \ln\left(\frac{\mathrm{A}(f_c)}{LB_m}\right)}, \qquad (3)$$

where LB_m is the link budget of the SF_m defined as $LB_m = P_{\max} - \theta_{rx_m}$, given the receiver sensitivity of each SF_m θ_{rx_m} in Table I. Hence, larger SFs result in larger communication ranges, with $l_{12} = R$.

If there is only one device n assigned to SF_m, this device is only subject to inter-SF interferences caused by devices using a different SF. Hence the inter-SF Signal-to-Interference-plus-Noise-Ratio (SINR) of end-device n can be expressed as

$$\mathrm{SINR}_{nm}^{\mathrm{inter}} = \frac{\gamma_{nm}}{\sum\limits_{i \in \mathcal{N}_{-n}} \sum\limits_{j \in \mathcal{M}_{-m}} s_{ij} \gamma_{ij} + 1}, \qquad (4)$$

where $\mathcal{N}_{-n} = \mathcal{N} \backslash \{n\}$, $\mathcal{M}_{-m} = \mathcal{M} \backslash \{m\}$ and s_{ij} is a binary variable used to indicate whether n uses m,

$$s_{ij} = \begin{cases} 1, \text{if } i \text{ uses } j \\ 0, \text{otherwise.} \end{cases}$$

When there is more than one device assigned to a SF, these devices are subject to both inter-SF and co-SF interferences. However, as shown in [5–7], co-SF interferences are largely dominant over inter-SF interferences[1]. Therefore, the co-SF SINR of end-device n on SF_m is written as,

$$\mathrm{SINR}_{nm}^{\mathrm{co}} = \frac{\gamma_{nm}}{\sum\limits_{i \in \mathcal{N}_{-n}} s_{im} \gamma_{im} + 1}. \qquad (5)$$

In conformity to LoRaWAN standards, instantaneous CSI feedback is not assumed, unlike [8]. Hence, the SF allocation is performed every period of time, during which the long-term fading instance, i.e., path loss distance, can be assumed to be fixed. Therefore, the achievable uplink short-term average rate for device n at SF_m is given similarly to [7] by,

$$\tau_{nm} = R_m \times P_{\mathrm{cap}}^{(n,m)}, \qquad (6)$$

where R_m is the data bit-rate of the SF_m and $P_{\mathrm{cap}}^{(n,m)}$ the probability of successful transmission, which is analyzed in the following section.

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[1]LoRa chirps were especially designed to yield limited inter-SF interferences.

III. PROBLEM FORMULATION

In this section, we formulate the considered optimization problem. Since fairness will be crucial in LoRa systems, we strive to maximize the minimal uplink short-term average rate over devices and SFs, under co-SF and inter-SF interferences. We first derive the expression of $P_{\text{cap}}^{(n,m)}$. Assuming $N > 1$, there are two cases:

1) One device n at SF_m: device n is only subject to inter-SF interferences. The transmission can be successfully decoded if the node satisfies the *inter-SF* as well as the *reception* condition. In this case inter-SF interferences are more critical than the reception condition since there will always be inter-SF interferences for $N > 1$. Hence the probability of successful transmission can be written as,

$$P_{\text{cap}_{\text{iSF}}}^{(n,m)} = P\left(\text{SINR}_{nm}^{\text{inter}} \geq \tilde{\theta}_m\right), \tag{7}$$

where $\text{SINR}_{nm}^{\text{inter}}$ is given in (4) and $\tilde{\theta}_m$ is the inter-SF interference capture threshold for SF_m, defined in Table I. Using the random instantaneous SNR variables γ_{nm} for all (n, m) and marginalizing over them, we show in the Appendix with similar calculations as in [7] that (7) can be written as,

$$P_{\text{cap}_{\text{iSF}}}^{(n,m)} = e^{-\frac{\tilde{\theta}_m \sigma_c^2 r_n^\alpha}{A(f_c) P_{\max}}} \prod_{\substack{(i,j) \in \\ \mathcal{N}_{-n} \times \mathcal{M}_{-m}}} \frac{1}{\tilde{\theta}_m s_{ij}\left(\frac{r_n}{r_i}\right)^\alpha + 1}. \tag{8}$$

2) More than one device at SF_m: in this case, co-SF interferences largely dominate the inter-SF interferences as well as reception condition as explained in Section II. Therefore, the success probability is expressed as in [12],

$$P_{\text{cap}_{\text{coSF}}}^{(n,m)} = P\left(\text{SINR}_{nm}^{\text{co}} \geq \theta_{\text{co}}\right), \tag{9}$$

where $\text{SINR}_{nm}^{\text{co}}$ is given in (5) and θ_{co} is the co-SF capture threshold which is equal to 6dB for all SF_m [3, 12]. With similar calculations as in the Appendix, we obtain

$$P_{\text{cap}_{\text{coSF}}}^{(n,m)} = e^{-\frac{\theta_{\text{co}} \sigma_c^2 r_n^\alpha}{A(f_c) P_{\max}}} \prod_{i \in \mathcal{N}_{-n}} \frac{1}{\theta_{\text{co}} s_{im} \times \left(\frac{r_n}{r_i}\right)^\alpha + 1}. \tag{10}$$

Given the above analysis, we can now formulate our objective function as follows (for $N > 1$),

$$\max_{s_{nm}} \min_{\substack{(n,m) \in \\ \mathcal{N} \times \mathcal{M} \\ \text{s.t.} s_{nm} \neq 0}} s_{nm} R_m P_{\text{cap}}^{(n,m)}, \tag{11}$$

where the minimization is over the s_{nm} that are non-zero, and

$$P_{\text{cap}}^{(n,m)} = I\left(\sum_{k \in \mathcal{N}} s_{km} \geq 2\right) P_{\text{cap}_{\text{coSF}}}^{(n,m)} + I\left(\sum_{k \in \mathcal{N}} s_{km} = 1\right) P_{\text{cap}_{\text{iSF}}}^{(n,m)}, \tag{12}$$

where $I(C)$ is the indicator function, i.e., it equals 1 if the condition C is verified and 0 otherwise.

Finally, the overall optimization problem becomes

$$(P) \max_{s_{nm}} \min_{\substack{(n,m) \in \\ \mathcal{N} \times \mathcal{M} \\ \text{s.t.} s_{nm} \neq 0}} s_{nm} R_m \left[I\left(\sum_{k \in \mathcal{N}} s_{km} \geq 2\right) e^{-\frac{\theta_{\text{co}} \sigma_c^2 r_n^\alpha}{A(f_c) P_{\max}}} \right.$$
$$\times \prod_{i \in \mathcal{N}_{-n}} \frac{1}{\theta_{\text{co}} s_{im}\left(\frac{r_n}{r_i}\right)^\alpha + 1} + I\left(\sum_{k \in \mathcal{N}} s_{km} = 1\right) e^{-\frac{\tilde{\theta}_m \sigma_c^2 r_n^\alpha}{A(f_c) P_{\max}}}$$
$$\left. \times \prod_{\substack{(i,j) \in \\ \mathcal{N}_{-n} \times \mathcal{M}_{-m}}} \frac{1}{\tilde{\theta}_m s_{ij}\left(\frac{r_n}{r_i}\right)^\alpha + 1} \right] \tag{13}$$

s.t.

$$\text{C1:} \quad s_{nm} \in \{0,1\} \quad \forall (n,m) \in \mathcal{N} \times \mathcal{M} \tag{13a}$$

$$\text{C2:} \quad \sum_{m \in \mathcal{M}} s_{nm} \leq 1, \forall n \in \mathcal{N} \tag{13b}$$

$$\text{C3:} \quad \sum_{n \in \mathcal{N}} s_{nm} \leq \text{N}_{\max}(m), \forall m \in \mathcal{M} \tag{13c}$$

$$\text{C4:} \quad \text{if } N > M, 1 \leq \sum_{n \in \mathcal{M}} s_{nm}, \forall m \tag{13d}$$

This optimization problem is constrained by the binary allocation variable $s_{nm} \in \{0,1\}$ (13a), the fact that an end-device n is assigned to at most one SF (13b) and that the maximal number of devices at SF_m is upper bounded by $\text{N}_{\max}(m)$ (13c)[2], if there are enough devices ($N > M$), no SFs should remain unused (13d). (P) is hence an integer programming problem, given the binary variables s_{nm}, with a non-linear objective function. This is an NP-hard problem [13], hence it is difficult to obtain its optimal solution directly. Therefore, we propose an optimized SF allocation method, using tools from matching theory.

IV. PROPOSED METHOD

Matching theory is a promising tool for resource allocation in wireless networks [14]. According to this theory, our considered allocation problem (P) can be classified as a many-to-one matching problem with conventional externalities and peer effects. There are two sets of players, the set of SFs and the set of end-devices, where each element seeks to be matched with players of the opposing set. A device will prefer to be matched to the SF offering the highest utility, while each SF prefers to be matched with the group of devices with the highest utility. The difficulty of our problem is that there is an interdependence between nodes' preferences, i.e., whenever a device is matched to an SF, the preferences of the other devices may change due to co-SF interferences. In addition to these conventional externalities (preference interdependence) and unlike the problem in [8] where only orthogonal channels (not SFs) were considered, our problem exhibits peer effects that are caused by inter-SF interferences. That is, the preferences of a device depend not only on the identity of the SF and the number of devices assigned to it, but also on the assignment of devices to other SFs (since they cause inter-SF interferences). Hence, to solve (P), we propose a many-to-one matching

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[2]Setting $\text{N}_{\max}(m)$ enables to control the harmful effects of co-SF interferences, and reduces the computational complexity of the proposed method, as shown in Sections IV-B and V-C.

algorithm between the set of SFs $\mathcal{M}$ and the set of end-devices $\mathcal{N}$. First, we define the basic concepts of matching theory.

Matching pair: a couple (n, m) assigned to each other.

Quotas of a player: the maximum number of players with which it can be matched

- Each end-device has a quota of 1 (13b),
- Each SF_m has a quota of $\mathrm{N_{max}}(m)$ devices (13c).

Utility of a node: defined for our problem as its short-term average rate. If it is the only device at SF_m,

$$U_n = R_m \mathrm{e}^{-\frac{\tilde{\theta}_m \sigma_c^2 r_n^\alpha}{\mathrm{A}(f_c) P_{\max}}} \prod_{\substack{(i,j) \in \\ \mathcal{N}_{-n} \times \mathcal{M}_{-m}}} \frac{1}{\tilde{\theta}_m s_{ij} \left(\frac{r_n}{r_i}\right)^\alpha + 1}. \quad (14)$$

If it shares the SF_m with other devices,

$$U_n = R_m \mathrm{e}^{-\frac{\theta_{\mathrm{co}} \sigma_c^2 r_n^\alpha}{\mathrm{A}(f_c) P_{\max}}} \prod_{i \in \mathcal{N}_{-n}} \frac{1}{\theta_{\mathrm{co}} s_{im} \left(\frac{r_n}{r_i}\right)^\alpha + 1}. \quad (15)$$

Utility of an SF: defined for our problem as the minimum short-term average rate among the devices assigned to it. If SF_m is matched to one device only:

$$U_m = \min_{n \in \mathcal{N}_m^*} R_m \mathrm{e}^{-\frac{\tilde{\theta}_m \sigma_c^2 r_n^\alpha}{\mathrm{A}(f_c) P_{\max}}} \prod_{\substack{(i,j) \in \\ \mathcal{N}_{-n} \times \mathcal{M}_{-m}}} \frac{1}{\tilde{\theta}_m s_{ij} \left(\frac{r_n}{r_i}\right)^\alpha + 1}, \quad (16)$$

where $\mathcal{N}_m^*$ is the set of devices assigned to the SF_m, otherwise U_m is given as

$$U_m = \min_{n \in \mathcal{N}_m^*} R_m \mathrm{e}^{-\frac{\theta_{\mathrm{co}} \sigma_c^2 r_n^\alpha}{\mathrm{A}(f_c) P_{\max}}} \prod_{i \in \mathcal{N}_{-n}} \frac{1}{\theta_{\mathrm{co}} s_{im} \left(\frac{r_n}{r_i}\right)^\alpha + 1}. \quad (17)$$

Preference relation: a player q prefers a player p_1 over the player p_2, if the utility of q is higher when it is matched to p_1 than when it is matched to p_2.

Blocking pair: a matching pair (n, m) is a blocking pair when U_n or U_m is higher when n uses m, than when they use their current matches, without lowering the utilities of any other device nor SF. In this case, n will leave its current match to be matched to m.

Two-sided stable matching: a matching solution where there is no blocking pair.

A. Proposed algorithm

In this subsection, we describe the steps of the proposed matching-based algorithm which exploits matching techniques as in [8, 14], tailored to our specific problem. First, the scheduler located at the gateway performs an initial matching between the set of SFs M and the set of end-devices N by the Initial Matching in Algorithm 1. Next, it swaps the matching pairs obtained in the previous step until reaching a two-sided exchange stable matching by Matching Refinement in Algorithm 2. Details of these steps are given below.

Let $\mathcal{L}_U$ denote the set of end-devices that are not allocated to any SF, req_m the requests received by the SF_m, and $\mathcal{A}_m$ the set of devices assigned to the SF_m. We suppose that the gateway knows the distance of all end-devices to it.

Initialization: the gateway starts by initializing the preference lists of end-devices and SFs. Each device n with a distance r_n to the gateway, can only use SFs including it in their coverage area ($r_n \leq l_m$), therefore,

$$\mathcal{L}_{p,n} = \{m \in \mathcal{M}, \text{ st } r_n \leq l_m\}, \quad (18)$$

$\mathcal{L}_{p,n}$ is arranged according to the increasing order of the distance threshold of the SFs ($l_m, m \in M$), i.e, an SF with higher achievable rate is preferred. On the other hand, an SF_m only considers devices having a distance to the gateway lower than l_m,

$$\mathcal{L}_{p,m} = \{n \in \mathcal{N}, \text{ st } r_n \leq l_m\}. \quad (19)$$

$\mathcal{L}_{p,m}$ is ordered such that, a user $n_1 \in \mathcal{L}_{p,m}$ is ranked before another user $n_2 \in \mathcal{L}_{p,m}$ if n_1 is located in the ring of SF_m ($n_1 \in (l_{m-1}, l_m]$) but not n_2 ($n_2 \notin (l_{m-1}, l_m]$), or both are in the ring of SF_m but n_1 is closer to the gateway than n_2 ($|l_{m-1} - r_{n_1}| < |l_{m-1} - r_{n_2}|$).

Unmatched end-devices are added to $\mathcal{L}_U$.

Initial Matching: for each device n in the unmatched list $\mathcal{L}_U$, if $\mathcal{L}_{p,n} \neq \emptyset$, n requests its first preferred SF and removes it from $\mathcal{L}_{p,n}$, otherwise the device is removed from $\mathcal{L}_U$ since all SFs it can use have already reached their quota. Then, each SF_m either accepts all current requests if its quota allows it, or it accepts the requests of its most preferred devices that fulfill its quota, if not. This process is repeated until $\mathcal{L}_U$ becomes empty. Note that, setting this quota of $N_{max}(m)$ devices for each SF_m also avoids large contention delays.

Matching Refinement: for each matching pair (n, m) the algorithm calculates U_m using (16) if it is only assigned to device n and (17) in the other case. The utility of device n is calculated by (14) if it is the only one at SF_m, and with (15) otherwise. Firstly, if there is an SF_l that is not assigned to any device that allows increasing U_n, the device leaves SF_m to be matched with SF_l. Then, the algorithm calculates the utilities of every pair (k, l), and makes a swap between (n, m) and (k, l) and determines their new utilities. If (k, m) or (n, l) is a blocking pair the algorithm validates the swap. This swapping step is repeated until reaching a two-sided stable matching.

B. Algorithm analysis

We now prove the stability and convergence of the proposed algorithm, and analyze its computational complexity.

Proposition 1. *Stability*: *When the proposed algorithm terminates, it finds a two-sided exchange stable matching.*

Proof. Let's assume that the proposed algorithm terminates and the final matching is not two-sided exchange stable. Then, the matching contains at least one more blocking pair (k, m) or (n, l) where the utility of at least one player among $\{n, m, k, l\}$, can be improved without lowering others' utility. Accordingly, the proposed algorithm did not terminate and will continue, thereby the matching is not final, which contradicts the initial assumption. □

Algorithm 1 Initial Matching

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while $\mathcal{L}_U$ *is not empty* **do**
  for $i \in \mathcal{L}_U$ **do**
    if $\mathcal{L}_{p,i} == \emptyset$ **then**
      $\mathcal{L}_U \leftarrow \mathcal{L}_U \backslash \{i\}$;
    else
      $a \leftarrow \text{firstPrefered}(\mathcal{L}_{p,i})$; // Favorite SF
      $\mathcal{L}_{p,i} \leftarrow \mathcal{L}_{p,i} \backslash \{a\}$;
      $\text{req}_a \leftarrow \text{req}_a \cup \{i\}$;

  for $j \in \mathcal{M}$ **do**
    if $size(\text{req}_j) > 0$ *and* $size(\mathcal{A}_j) < \text{N}_{\text{Max}}(j)$ **then**
      if $(size(\text{req}_j) + size(\mathcal{A}_j)) \le \text{N}_{\text{Max}}(j)$ **then**
        Accept all the requests and add the devices to $\mathcal{A}_j$;
      else
        Accept the requests of the $\left(\text{N}_{\text{Max}}(j)\text{-}size(\mathcal{A}_j)\right)$ *most preferred devices, and add them to* $\mathcal{A}_j$;

Algorithm 2 Matching Refinement

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while change == true **do**
  change = false;
  for $j \in \mathcal{M}$ **do**
    Calculate U_j *eq (16) or eq (17)*;
    for $i \in \mathcal{A}_j$ **do**
      Calculate U_i *eq (14) or eq (15)*;
      for $l \in \mathcal{M}_{-j}$ **do**
        if $size(\mathcal{A}_l) == 0$ **then**
          $\text{Swap}\left((i,j),(\emptyset,l)\right)$;
          Calculate the new utility of i, U'_i *eq (14) or eq (15)* ;
          if $\text{U}'_i \ge \text{U}_i$ **then**
            Validate the Swap;
            change $\leftarrow$ true;
        else
          Calculate U_l *eq (16) or eq (17)*;
          for $k \in \mathcal{A}_l$ **do**
            Calculate U_k *eq (14) or eq (15)*;
            $\text{Swap}\left((i,j),(k,l)\right)$;
            if *(i,l) or (k,j) is a blocking pair* **then**
              Validate the Swap;
              change $\leftarrow$ true;

Proposition 2. ***Convergence***: *After a finite number of swap operations, the algorithm eventually converges to a two-sided exchange stable matching.*

Proof. A swap operation occurs if it improves the utility of at least one player without decreasing the others', hence the utilities can only rise. Additionally, the maximal throughput that can be achieved on an SF_m is upper-bounded by the data bit-rate R_m, meaning that each SF_m and the devices assigned to it have utilities upper bounded by R_m.
The number of potential swap operations is finite: device assigned to SF_l can make at most $\text{N}_{\text{max}}(l) \times \sum_{j \in \mathcal{M}_{-l}} \text{N}_{\text{max}}(j)$ swap operations. The total number of swap operations is thus upper-bounded by $\sum_{l \in \mathcal{M}} \text{N}_{\text{max}}(l) \times \sum_{j \in \mathcal{M}_{-l}} \text{N}_{\text{max}}(j)$. $\qquad\square$

Proposition 3. ***Complexity***: *The running time of our proposed algorithm is upper-bounded by* $\mathcal{O}\left(NM + Q^2 M^2\right)$.

Proof. Initial matching complexity: in the worst case, all the devices have the same preference list, and they are located in the area covered by all the SFs. At round_1 the gateway receives N requests, at round_2 it receives $N - \text{N}_{\text{max}}(m_1)$ requests, at round_i it receives $N - \sum_{k=1}^{M-1} \text{N}_{\text{max}}(m_i)$ requests. Therefore, the total number of requests equals $NM - \sum_{i=1}^{M-1} (M-i) \times \text{N}_{\text{max}}(m_i)$. The complexity of the initial matching is upper bounded by: $\mathcal{O}\left(NM\right)$.

Matching refinement complexity: in each iteration, for each SF_m, the algorithm considers at most $\text{N}_{\text{max}}(m)$ devices and examines $\sum_{l \in \mathcal{M}_{-m}} \text{N}_{\text{max}}(l)$ swap operations for each of these devices. Therefore, the number of swap operations that are examined in one iteration is upper bounded by $\sum_{m \in \mathcal{M}} \text{N}_{\text{max}}(m) \times \sum_{l \in \mathcal{M}_{-m}} \text{N}_{\text{max}}(l)$. Let $Q = \max_{m \in \mathcal{M}} \{\text{N}_{\text{max}}(m)\}$, thus the computational complexity of the matching refinement is upper bounded by $\mathcal{O}\left(Q^2 M(M-1)\right)$.

In summary, the computational complexity of our algorithm is upper bounded by $\mathcal{O}\left(NM + Q^2 M^2\right)$. $\qquad\square$

Note that this complexity is not excessive as our algorithm is run at the gateway which is not computationally-limited.

V. NUMERICAL RESULTS

A. Simulation Settings

We consider a cell of radius R=1km [12], with a varying number of devices N from N=2 to 40. Note that all devices transmit with a duty cycle of 100%. Hence, with a duty cycle of 1% as preconized in LoRaWAN [1], the actual number of end-devices would be 100-fold[3], i.e., up to 4000. All end-devices transmit in the channel of carrier frequency f_c=868[MHz] with a bandwidth BW=125[kHz] and power $P_{\text{max}} = 14$ [dBm]. We consider a lossy urban environment, therefore we take a path loss exponent equal to 4 as in [7, 12]. The computer simulation environment has been implemented in Matlab[4].

B. Baseline schemes

We consider two baseline schemes for performance comparison the random SF allocation [7], and the distance-SF allocation algorithms [11], with a maximal number of simultaneously transmitting devices equal to A (specified later):
Random SF: the gateway chooses randomly A devices among $\mathcal{N}$ and assigns a random SF to each of these devices among the possible SFs.
Distance SF: the gateway chooses randomly A devices among $\mathcal{N}$. Then, the SF for each of these devices is determined by

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[3]The evaluations are made for 100% duty cycle as this is the most challenging case. Hence, much better performance can be expected in the case of 1% duty cycle.

[4]All results are based on our mathematical model detailed in the Appendix and whose validity has been proved in [7].

N_{Max}	SF7	SF8	SF9	SF10	SF11	SF12
1	4.82	1.51	1.06	4.7e-1	2.7e-1	1.9e-1
2	7.7e-2	1.1e-7	9.3e-14	7.8e-25	6.7e-46	3.7e-78
3	2.7e-3	8.2e-9	2e-15	8.2e-27	3.1e-49	1.3e-84
4	9.9e-5	5.8e-10	9.0e-17	4.3e-29	1.2e-49	1.1e-86
5	1.8e-6	5.2e-11	6.5e-18	1.3e-30	1.0e-53	3.7e-93

TABLE II
MINIMAL THROUGHPUT FOR EACH SF_m [KBIT/S]

Table I based on their distance r_n: device n uses SF_m if $r_n \in (l_{m-1}, l_m]$.

C. Choice of $\mathrm{N}_{\max}$ *given a target minimum throughput*

To determine the quota of each SF, we fix a target minimal throughput equal to 1 [bit/s]. We have run preliminary simulations over 100000 frames. Table II represents the minimal short-term average rate achieved on each SF, for different values of $\mathrm{N}_{\max}$. We can observe that to guarantee the target minimal throughput of 1 [bit/s], we can have at most three devices assigned to SF_7 but only one device for the other SFs. In the sequel, the proposed method is compared to the baseline SF-allocation schemes. For fair comparison, the number of simultaneously transmitting devices A is equal to $\sum_{m \in \mathcal{M}} \mathrm{N}_{\max}(m) = 8$ in each allocation period.
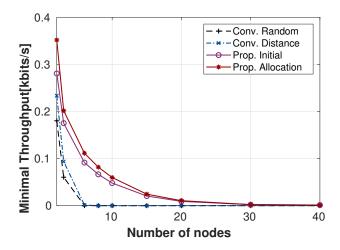
D. Simulation results

Fig. 2. Minimal short-term average rates, proposed and baseline algorithms

Figure 2 depicts the performance comparison of our SF-allocation algorithm, and the baseline schemes in terms of minimal short-term average rates, against a varying number of end-devices. First, we can observe that the proposed allocation outperforms both the conventional random and distance allocations for all values of N. While baseline schemes lead to an early drop of minimal rate (almost null for $N > 6$), the proposed algorithms still provide a good minimal throughput for a much higher number of devices. We can also notice that our Matching Refinement (Prop. Allocation) provides a better minimal throughput than the Initial Matching (Prop. Initial).

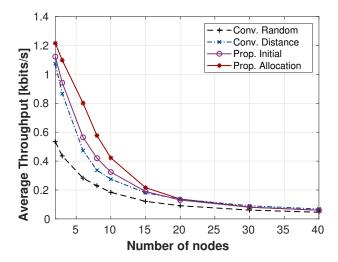Next, Figure 3 shows the impact of maximizing the minimal

Fig. 3. Average network throughput, proposed and baseline algorithms

short-term average rates on the average network throughput over all end-devices. It can be clearly seen that the highest average network throughput is achieved by our proposed SF-allocation method. Also, it can be noted that our Matching Refinement provides a significant improvement compared to the Initial Matching.

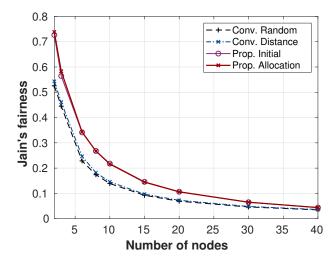Finally, we evaluate the fairness levels of the different algo-

Fig. 4. Jain's fairness metric, proposed and baseline algorithms

rithms by the Jain's fairness index, given by $\mathcal{J} = \frac{\left(\sum_{n \in \mathcal{N}} U_n \right)^2}{N \times \sum_{n \in \mathcal{N}} U_n^2}$.

Figure 4 shows that by the considered max-min strategy and matching-based methodology, the proposed algorithms significantly enhance the system fairness level compared to baseline methods.

Overall, the proposed SF-allocation method provides remarkable performance improvements, jointly in terms of minimal achievable rates, network throughput and fairness, with a

limited computational complexity suitable for LoRa gateways.

VI. Conclusion

We have considered the problem of fairness-aware SF-allocation in a LoRa-based network composed of IoT end-devices. Unlike most previous works, we have integrated both the effects of co-SF and inter-SF interferences in the expression of the achievable uplink short-term average rate of each end-device. We have then formulated our SF allocation problem as a max-min optimization of the uplink short-term average rate of end-devices. To solve this NP-hard optimization problem, we have proposed an SF-allocation algorithm based on matching theory, and proved its stability, convergence properties and its reasonable complexity. Numerical results show that our SF-allocation strategy not only outperforms the conventional algorithms in terms of minimal rate, but also jointly in terms of network throughput and fairness. For future work, this problem will be extended by jointly controlling the transmission power and SF allocation and by designing distributed algorithms. Energy efficiency aspects will be also investigated.

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Appendix

We now prove the expression (8). Using the random instantaneous SNR variables γ_{nm} for all (n,m), (7) can be developed as

$$P\left(\frac{\gamma_{nm}}{\sum\limits_{\substack{(i,j)\in \\ \mathcal{N}_{-n}\times\mathcal{M}_{-m}}} s_{ij}\gamma_{ij}+1} \geq \tilde{\theta}_m \middle| \gamma_{11},\ldots\gamma_{n-1m-1},\gamma_{n+1m+1},\ldots\gamma_{NM}\right)$$
$$\times P\left(\gamma_{11},\ldots\gamma_{n-1m-1},\gamma_{n+1m+1},\ldots\gamma_{NM}\right). \quad (20)$$

The variables γ_{nm} are modeled as exponential random variables with mean $\overline{\gamma}_{nm}$. Therefore, by marginalizing over these variables we obtain

$$\int\limits_{\gamma_{11}}\cdots\int\limits_{\gamma_{n-1m-1}}\int\limits_{\gamma_{n+1m+1}}\cdots\int\limits_{\gamma_{NM}} \underbrace{P\left(\gamma_{nm}\geq\tilde{\theta}_m\left(\sum\limits_{\substack{(i,j)\in \\ \mathcal{N}_{-n}\times\mathcal{M}_{-m}}} s_{ij}\gamma_{ij}+1\right)\right)}_{A}$$
$$\times P\left(\gamma_{11},\ldots\gamma_{n-1m-1},\gamma_{n+1m+1},\ldots\gamma_{NM}\right)\,\mathrm{d}\gamma_{11}\cdots\mathrm{d}\gamma_{NM}. \quad (21)$$

First, we calculate A where $p_\gamma(\gamma_{nm})=\frac{1}{\overline{\gamma}_{nm}}\mathrm{e}^{-\frac{\gamma_{nm}}{\overline{\gamma}_{nm}}}$

$$A = \int\limits_{\tilde{\theta}_m\left(\sum\limits_{(i,j)\in\mathcal{N}_{-n}\times\mathcal{M}_{-m}} s_{ij}\gamma_{ij}+1\right)}^{\infty} p_\gamma(\gamma_{nm})\,\mathrm{d}\gamma_{nm}$$
$$= \mathrm{e}^{-\frac{\tilde{\theta}_m}{\overline{\gamma}_{nm}}}\prod\limits_{(i,j)\in\mathcal{N}_{-n}\times\mathcal{M}_{-m}}\frac{1}{\frac{\tilde{\theta}_m s_{ij}\overline{\gamma}_{ij}}{\overline{\gamma}_{nm}}+1}. \quad (22)$$

By replacing A in $P_{\mathrm{cap_{iSF}}}^{(n,m)}$, we have

$$P_{\mathrm{cap_{iSF}}}^{(n,m)} = \mathrm{e}^{-\frac{\tilde{\theta}_m}{\overline{\gamma}_{nm}}}\prod\limits_{\substack{(i,j)\in \\ \mathcal{N}_{-n}\times\mathcal{M}_{-m}}}\int\limits_{\gamma_{ij}}\mathrm{e}^{-\frac{\tilde{\theta}_m s_{ij}\gamma_{ij}}{\overline{\gamma}_{nm}}}p_\gamma\left(\gamma_{ij}\right)\,\mathrm{d}\gamma_{ij}$$
$$= \mathrm{e}^{-\frac{\tilde{\theta}_m}{\overline{\gamma}_{nm}}}\prod\limits_{\substack{(i,j)\in \\ \mathcal{N}_{-n}\times\mathcal{M}_{-m}}}\frac{1}{\frac{\tilde{\theta}_m s_{ij}\overline{\gamma}_{ij}}{\overline{\gamma}_{nm}}+1}. \quad (23)$$

Finally, we obtain

$$P_{\mathrm{cap_{iSF}}}^{(n,m)} = \mathrm{e}^{-\frac{\tilde{\theta}_m\sigma_c^2 r_n^\alpha}{\mathrm{A}(f_c)P_{\max}}}\prod\limits_{\substack{(i,j)\in \\ \mathcal{N}_{-n}\times\mathcal{M}_{-m}}}\frac{1}{\tilde{\theta}_m s_{ij}(\frac{r_n}{r_i})^\alpha+1}. \quad (24)$$