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Computation of Ground Surface Conduction Heat Flux by Fourier Analysis of Surface Temperature

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ABSTRACT

A method for computing the ground surface heat flux density is tested at two places in West Africa during the rainy season and during the dry season. This method is based upon the Fourier analysis of the experimental ground surface temperature. The only required parameter is the soil thermal inertia. The results of these calculations agree with the measurements. This method avoids the use of empirical formulas relating the ground heat flux density to other terms of the surface energy budget. It is shown that these relations are not universal.

1. Introduction

The ground surface energy budget is expressed by the equation

$$R_n = H + LE + G_0, \tag{1}$$

where the various terms are flux densities for net radiation (R_n), latent heat (LE), sensible heat (H) and ground surface heat conduction (G_0). For the estimation of diurnal variations of G_0 when it is not measured, empirical formulas are used which express G_0 as a function of other terms of the energy budget. For a meadow in the Netherlands, the relation

$$G_0 = 0.1R_n \tag{2}$$

was established by De Bruin et al. (1982). For a rural site in Sweden, De Heer et al. (1981) obtained the relation

$$G_0 = 0.356R_n - 21. \tag{3}$$

From experiments on logged sites in Canada, MacCaughey (1982) estimated G_0 between 1 and 15 percent of R_n, according to the nature of the ground. At lower latitudes, in Arizona, Idso et al. (1975) obtained G_0 from a regression with R_n

$$G_0 = 0.316R_n - 59.9 \tag{4a}$$

$$G_0 = 0.507R_n - 48.8 \tag{4b}$$

where (4a) is for wet ground and (4b) is for dry ground. Thus, there is a great diversity in the parameterization of G_0 as a function of R_n.

Such formulas are very useful at sites that have been previously checked. On the other hand, for a site where no test is available, they may not be applicable. In the case of tropical regions, errors on G_0 may even be quite drastic, as this flux becomes very large (except for strongly vegetated surfaces), reaching frequently 150 W m^{-2} (Coulibaly, 1981; Druilhet et al., 1981; Durand, 1983).

2. Methods for obtaining G_0

All the fluxes mentioned here are expressed in W m^{-2}. They are averaged over a time period between 20 minutes and 1 hour.

a. Measurements

The G_0 may be measured either directly, using a fluxmeter, or indirectly with temperature probes set into the ground. The fluxmeter is easy to use provided that a careful calibration can be performed (Schwerdtfeger, 1976), but it smooths the rapid fluctuations of the flux (Druilhet et al., 1981). Another way is to set thermometers into the ground from the surface to a depth of about 1 m. The vertical separation of the probes should follow a geometrical progression (Viton, 1970). Moreover, the specific heat C of the ground must be known. Then

$$G_0 = \int_0^{z_{\text{inf}}} C(\partial T/\partial t)\, dz, \tag{5}$$

where $T(z, t)$ is the ground temperature at depth z and time t, and z_{inf} is the depth of the lower level, at which G_0 is assumed to vanish.

b. Empirical formulas

When no measured value is available for G_0, it may be evaluated as a function of other terms of (1). As seen in the Introduction, G_0 may be expressed as a function of R_n. Another type of parameterization estimates G_0 from the values of the sensible heat flux density H. The relation

$$G_0 = \alpha H \tag{6}$$

was proposed by Kasahara and Washington (1971), with a value of ⅓ for α, and used in a general circulation model. However, this value is questioned by Berkowicz and Prahm (1982), who showed that the ratio G_0/H can be notably different from the value ⅓, indeed varying between ⅓ and 1, depending on the site characteristics.

3. Fourier analysis of surface temperature

a. Method

The resolution of the heat conduction equation for a semi-infinite homogeneous solid with constant physical properties, when its plane boundary is submitted to a periodic variation of temperature, gives, at the order k,

$$T(z, t) = \theta_0 + \sum_{n=1}^{k} \exp[-(n\omega/2\kappa)^{1/2} z]\left\{a_n \cos\left[n\omega t - \left(\frac{n\omega}{2\kappa}\right)^{1/2} z\right] + b_n \sin\left[n\omega t - \left(\frac{n\omega}{2\kappa}\right)^{1/2} z\right]\right\}. \tag{7}$$

Such an expansion can be applied to the surface temperature. The ground physical properties are assumed homogeneous and constant. Then, θ_0, a_n and b_n are the Fourier coefficients for the diurnal wave of temperature; $\omega = 2\pi/\tau$, where τ is the diurnal period (24 h); κ is the ground thermal diffusivity (Carslaw and Jaeger, 1978), and G_0 is defined by

$$G_0 = -\lambda(\partial T/\partial z)_{z=0}, \tag{8}$$

where λ is the ground thermal conductivity. Then, at order k, we have:

$$G_0 = I(\omega)^{1/2} \sum_{n=1}^{k} n^{1/2}[a_n \cos(n\omega t + \pi/4) + b_n \sin(n\omega t + \pi/4)], \tag{9}$$

where I is the thermal inertia

$$I = (C\lambda)^{1/2}. \tag{10}$$

TABLE 1. Effect on G_0 computation of a random error in surface temperature (ECLATS, 8 December).

	ΔT		
	0 (undisturbed case)	2 (°C)	4 (°C)
r	0.99	0.98	0.97
a	0.99	0.98	0.95
b	10.2	9.5	9.2
δ (%)	4.7	6.9	9.4

TABLE 2. Effect on G_0 calculation of a step error in surface temperature due to sun exposure (ECLATS, 8 December).

	θ_e		
	0 (undisturbed case)	2 (°C)	−2 (°C)
r	0.99	0.99	0.99
a	0.99	1.04	0.93
b	10.2	10.6	9.4
δ (%)	4.7	6.7	7.2

Thus the determination of G_0 requires only the evaluation of Fourier coefficients of the surface temperature signal. The daily trend is generally weak and is extracted from the signal prior to the analysis.

A similar method was developed by Horton and Wieringa (1983), requiring two or three measurement levels of ground temperature, excluding the soil surface.

b. Experimental data

This method was tested using data from two experiments carried out in West Africa.

1) WAMEX—WEST AFRICAN MONSOON EXPERIMENT, 1979

Experimental data have been collected during WAMEX in 1979 by the Laboratoire Associé de Météorologie Physique (L.A.M.P., Université de Clermont-Ferrand II, France). The measurement site was located near Korhogo airport (9°30′N, 5°30′W, about 400 m MSL) on the Northern Ivory Coast. The ground surface was partly covered with grass (about 30 cm high). Wind speed, temperature and humidity profiles in the surface layer were measured using a 6 m high mast with three measurement levels: 0.5, 2 and 6 m. The instrumentation consisted of cup anemometers, dry and wet bulb thermometers, and a vane at the top of the mast. Six thermal probes measured the soil temperatures at depths: 0, 5, 10, 20, 40 and 80 cm. A Kipp pyranometer and a raingauge completed the instrument set. All of the data were recorded on a magnetic tape at 5 minute intervals. Twenty-five minute average values were calculated. The sensible and latent heat fluxes were computed using the aerodynamic method (Saugier and

TABLE 3. Comparison of computed and measured values of G_0 during WAMEX.

	1 Aug	2 Aug	3 Aug	4 Aug	6 Aug	7 Aug	8 Aug	9 Aug	10 Aug
r	0.85	0.89	0.81	0.88	0.83	0.83	0.79	0.76	0.78
a	1.01	1.13	1.03	0.99	0.98	0.99	1.03	0.98	0.91
b	−22.6	−28.5	9.0	−24.4	−13.5	−3.6	−1.3	0.32	−12.6
δ (%)	16.4	14.7	16.2	15.7	13.8	16.8	19.4	23.3	19.0

Ripley, 1978). The accuracy of this method is typically 15–25% for daytime values. The G_0 was computed by the soil temperature profile method, as described before by (5), with a value for C of $2 \cdot 10^6$ J kg^{-1} K^{-1}. Net radiation was derived from the global radiation measurements, using the Angström formula to compute atmospheric infrared radiation (Sutton, 1953). The terms of the energy budget are available with a 25-minute time step (Coulibaly, 1981; Cautenet et al., 1985).

2) ECLATS—ETUDE DE LA COUCHE LIMITE TROPICALE SÈCHE: DRY TROPICAL BOUNDARY LAYER EXPERIMENT, 1980

The 1980 ECLATS was jointly conducted by several French and African laboratories. The surface layer measurements were carried out by the Laboratoire d'Aérologie (Université de Toulouse, France) near Niamey airport (13°30'N, 2°10'E), in the Republic of Niger, during the dry season (November and December 1980). The surrounding vegetation consisted of dry thorn bushes and shrubs, but measurements were performed on a bare surface enclosure, using a station designed by the Institut National de la Recherche Agronomique. The measured parameters were net radiation, ground heat flux G_0, wind speed at levels 0.75 and 2.25 m, temperature gradient between these levels, and the ground surface temperature. The sensible heat flux density was evaluated using the aerodynamic simplified method developed by Itier (1980). The G_0 was measured with a fluxmeter and corrected by a calibration in situ using the soil temperature data, which also permitted the determination of the attenuation and phase of the higher order harmonics (Druilhet, 1983). The latent heat flux is very weak, due to soil dryness, and was not directly measured. The terms of the budget equation are available with a time step of 20 minutes (Druilhet et al., 1981; Druilhet et al., 1982; Durand, 1983).

4. Tests on the method

a. *Fittings of the order of expansion and of the thermal inertia*

The choice of the order k of the Fourier expansion was made according to the following observations: as the temperature signal is compared with the estimate given by a Fourier expansion, an important degradation may be observed for low values of k (say, 3 or 4), whereas for larger values of k (above 20), a numerical noise is likely to appear. An intermediate value of 10 was chosen in order to avoid such shortcomings. Moreover, this value of 10 corresponds to a maximum of the correlation coefficient between experimental values of G_0 and the values from (9).

As no direct measurement of thermal inertia was available, this parameter was determined as follows: for every day of both experiments, we calculated G_0 by (9) and the calculated curve was compared with the experimental one. The value of I was modified at every run so as to converge both curves as much as possible. This fit was controlled by use of an estimate of the distance between calculated and experimental values of G_0:

$$\delta = \frac{1}{M}\left[\frac{\sum (G_{0\ \mathrm{measured}} - G_{0\ \mathrm{calculated}})^2}{N}\right]^{1/2} \qquad (11)$$

where M is the daily range of G_0 (experimental values):

$$M = G_{0\ \mathrm{meas}}{}^{\mathrm{max}} - G_{0\ \mathrm{meas}}{}^{\mathrm{min}},$$

and N is the number of values of G_0 available every day (58 for WAMEX and 72 for ECLATS); δ is therefore the relative error of the method.

For every day, we could therefore select the best value of the thermal inertia. For a given place, these values

TABLE 4. Comparison of computed and measured values of G_0 during ECLATS.

	21 Nov	22 Nov	23 Nov	27 Nov	29 Nov	1 Dec	2 Dec	6 Dec	7 Dec	8 Dec
r	0.99	0.98	0.98	0.99	0.97	0.99	0.99	0.99	0.99	0.99
a	0.95	0.94	0.98	0.99	1.00	0.97	0.97	0.99	1.01	0.99
b	8.2	18.7	16.2	6.4	19.8	1.9	5.3	11.1	16.0	10.2
δ (%)	5.8	7.7	6.1	4.7	8.1	3.8	4.0	3.8	5.4	4.7

showed little variation, in spite of large variations in the meteorological conditions. The mean values of I are respectively 1700 S.I. units (J K^{-1} m^{-2} $s^{-1/2}$) for WAMEX, and 1000 S.I. for ECLATS. These mean values were used for the calculations presented here. On the other hand, the values of I are very different according to the site, which could be explained by the difference in the nature and water content of the soil constituting the surface layers. The higher values of I during WAMEX are explained by the fact that this experiment was carried out during the rainy season, whereas ECLATS took place during the dry season, since I is an increasing function of the water content in the soil (De Vries, 1966).

b. *Sensitivity to the surface temperature*

The comparison between measured and calculated values of G_0 was done using the following standard parameters:

r correlation coefficient;
a and b coefficients of the linear regression

$$G_{0\,\text{calculated}} = aG_{0\,\text{measured}} + b; \qquad (12)$$

δ relative distance between the measured and computed values of G_0 as described by (11).

With the described method, a constant bias off the true surface temperature (i.e., proceeding from inaccurate calibration of the thermometer) has no effect on the computed values of G_0. It may be seen from (7) and (9) that random errors in T will affect the calculation of G_0. A random error, for instance, fluctuations (noise) arising from the measuring instrument, was generated with values between $\pm\Delta T$ = 2–4°C (i.e., about 10% and 20% of the daily surface temperature range). Results are presented in Table 1 for the data obtained on 8 December 1980 (ECLATS). The correlation coefficient remains close to unity. The relative distance δ for

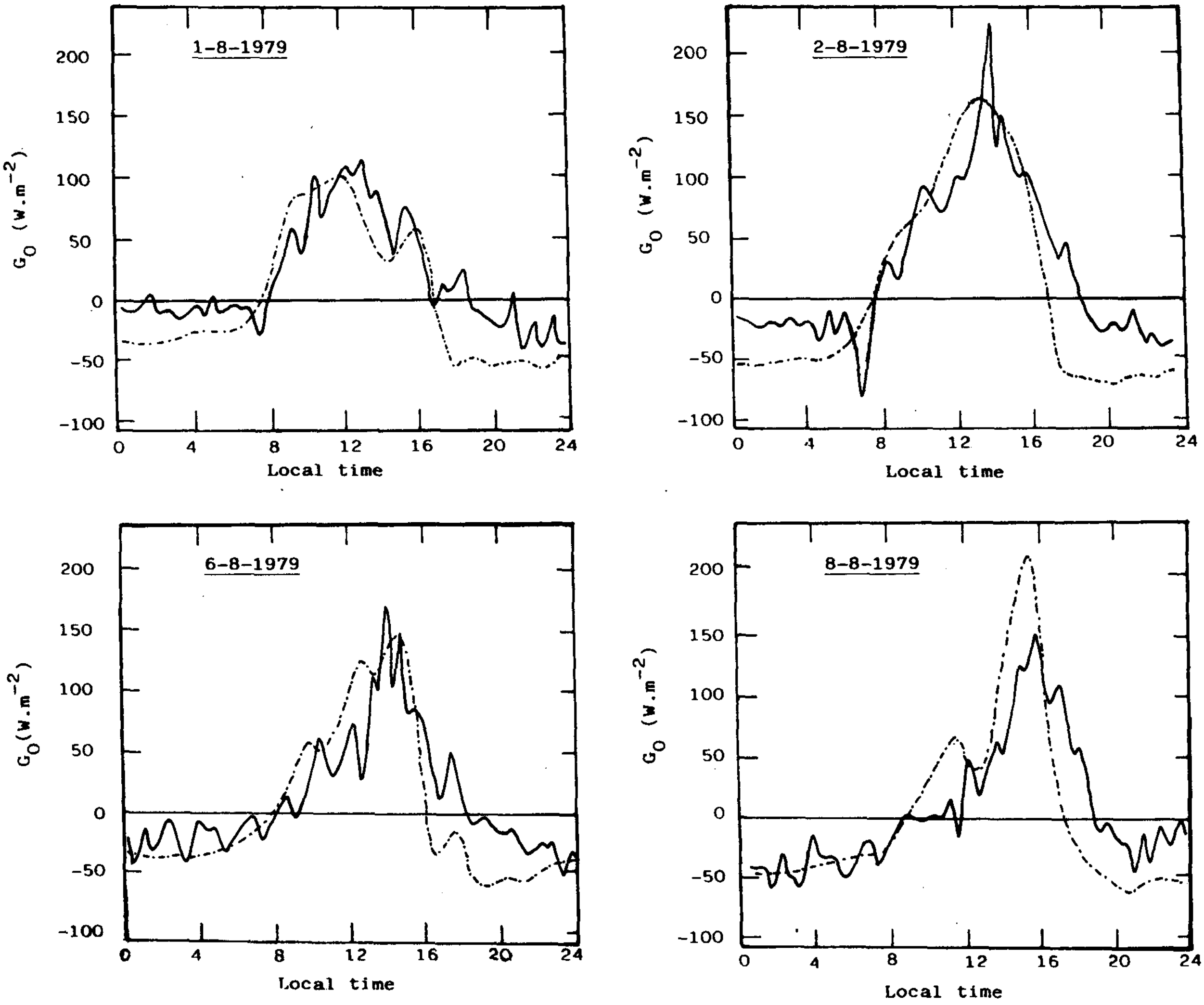

FIG. 1. Comparison of calculated values of G_0 with WAMEX data: calculated values (dashed line); experimental values (solid line).

$\Delta T = 4°\text{C}$ is doubled as compared with its undisturbed value ($\Delta T = 0$). However, its value still remains small.

Finally, a step error θ_e, simulating a constant bias of the probe response during sun exposure, was tested for values of $\theta_e = \pm 2°\text{C}$. This bias was assumed to grow from zero to θ_e during 1 hour after sunrise and to remain constant during daytime. After sunset, it decayed during 1 hour from θ_e to zero. The same day (8 December) was used for this test. The results are presented in Table 2; it shows that the correlation coefficient remains practically unchanged. The only noticeable effect is the modification of the parameter a, which is decreased by 7% for a decrease of 2°C of daytime temperature and increased by 4% for an increase of 2°C. Thus, ϵ deviation of the daytime surface temperature involves a deviation of G_0 in the same direction. In any case the modification of the estimate of G_0 is not important.

c. *Results*

The previously defined parameters r, a, b and δ are computed for a sample of arbitrarily chosen days for each experiment, and presented in Tables 3 (WAMEX) and 4 (ECLATS). For WAMEX, the values of r range between 0.76 and 0.89, and δ may vary between 14% and 23%. For ECLATS, r is close to unity and δ does not exceed 8%. This greater proximity between $G_{0\,\text{calculated}}$ and $G_{0\,\text{measured}}$ for ECLATS is probably due to the less variable meteorological conditions compared to WAMEX, a feature which appears clearly when comparing the curves of Figs. 1 (WAMEX) and 2 (ECLATS). In WAMEX, the variations of G_0 over a short period are not accurately estimated, with obvious shift between measured and calculated curves of G_0 at sunset, while the mean diurnal variations are well-reproduced. In ECLATS, the daytime computed values

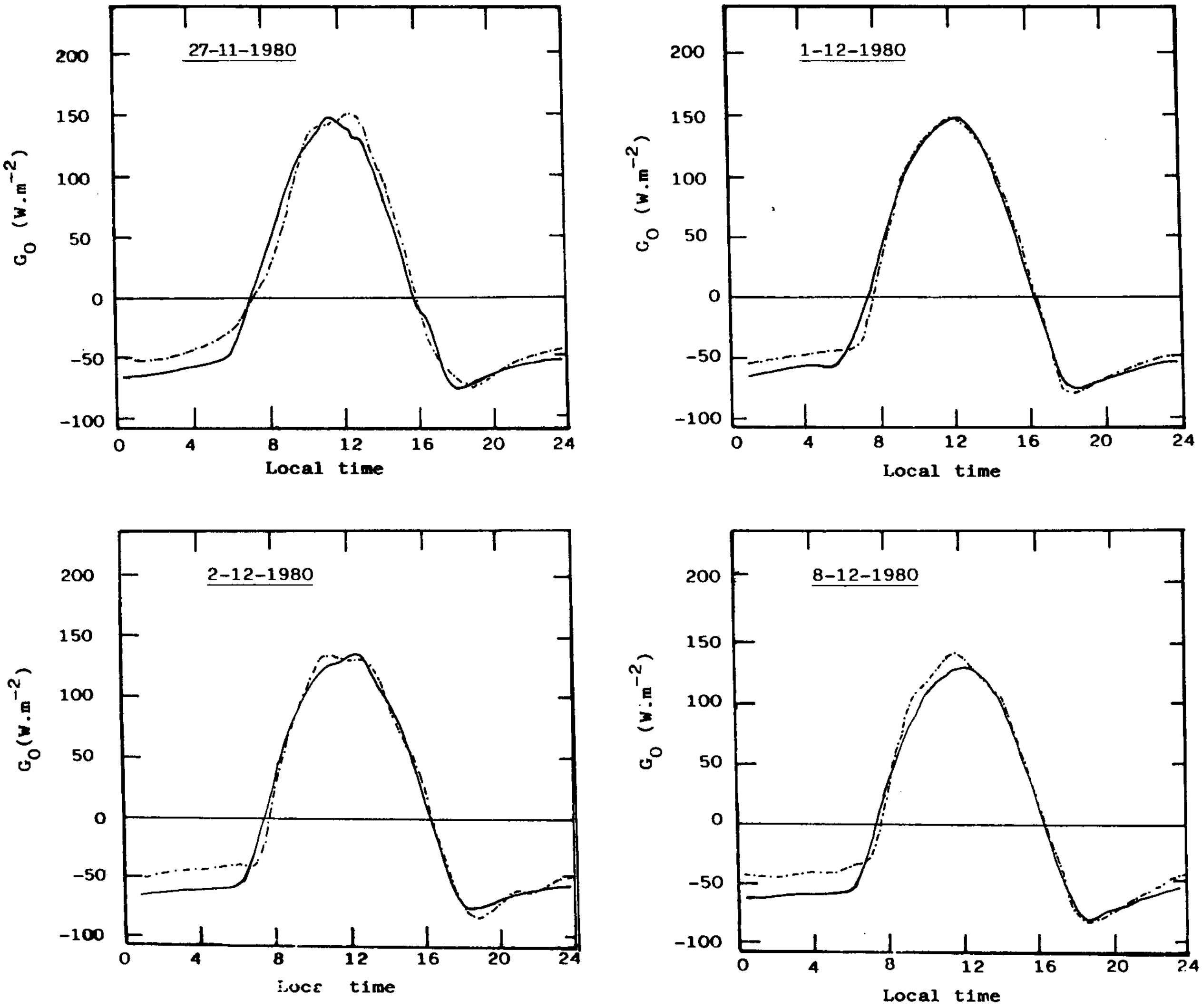

FIG. 2. Comparison of calculated values of G_0 with ECLATS data: calculated values (dashed line); experimental values (solid line).

of G_0 are very close to the experimental data. During nighttime, a systematic departure may be observed: the computed values are underestimated by about 20% or 30%. Generally, this departure begins at 2100 (local time). It increases during the night and vanishes after sunrise. A possible origin of this phenomenon could be the daily variation of water content (daytime drying by evaporation, nighttime restoration by capillarity from the inner soil layers). This water cycle induces a daily cycle of thermal inertia. Little information is available about thermal properties of soil in this region, but some data (Sellers, 1965) show that when the ground is very dry (volumetric moisture content less than 10%), its thermal inertia increases very quickly with increasing soil wetness. Thus, the true nighttime thermal inertia could be much greater than the assumed constant value used here. However, we cannot be more explicit as no measurements of ground wetness are available. On the other hand, for WAMEX, this phenomenon is not obvious, because with such wet ground the relative variations of thermal inertia with humidity are weak.

Nevertheless, this method seems more reliable than a determination of G_0 from empirical relations including another term of the surface heat budget. In order to illustrate possible shortcomings of such formulas, we present a diagram of G_0 plotted versus H (diurnal values, averaged over 25 min) in Fig. 3. There is no obvious relation between G_0 and H.

5. Conclusion

A method for computing ground surface heat density G_0 from surface temperature was tested using data from WAMEX (Korhogo, Ivory Coast, rainy season) and from ECLATS (Niamey, Niger, dry season). The main features of the obtained results are

- The Fourier expansion to order 10 of surface temperature provides the best estimate of G_0.
- The relative distances between measured and computed values of G_0 never exceed 8% for ECLATS and 23% WAMEX.
- The calculated value of G_0 is little changed by random or systematic errors on ground surface temperature.

This method eliminates any use of empirical formulation of G_0 as a function of other terms of the energy budget. It needs only the measurement of the surface temperature with probes such as thermistors or thermocouples.

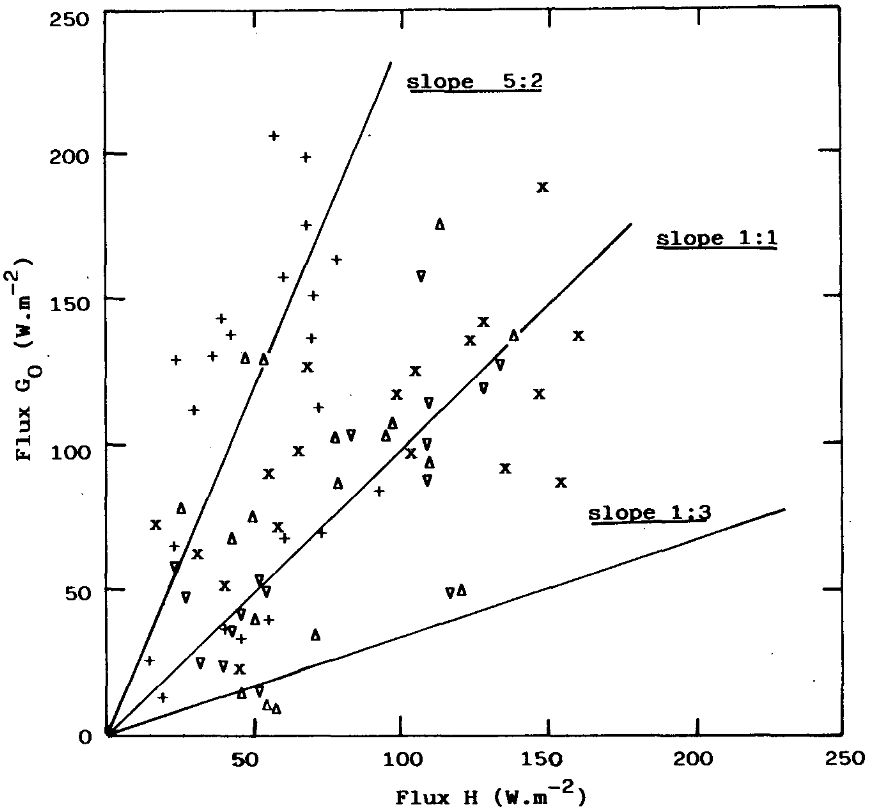

FIG. 3. Diagram of diurnal (10–16 h) values of G_0 and H, averaged over 25 mm (WAMEX data; 2 August 1979 (crosses); 4 August 1979 (pluses); 8 August 1979 (inverted triangles); 9 August 1979 (triangles).

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